14.03/003 Micro Theory & Public Policy, Fall 2025

Lecture slides 4. Consumer preference and the theory of choice

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Announcements

- **This Wednesday**
  - Read for in-class class discussion:

    Bleemer, Zachary, and Aashish Mehta. "Will studying economics make you rich? A regression discontinuity analysis of the returns to college major." *American Economic Journal: Applied Economics* 14, no. 2 (2022): 1-22
  - What's the question of the paper?
  - What's the main methodology for answering that question?
  - What are the key findings?
  - What questions does this leave you with
  - *Apologies: No Autor office hours this Weds*

- **Next Monday (9/22) — How much do low-income families value health insurance?**
  - Read for in-class discussion:

    Finkelstein, Amy, Nathaniel Hendren, and Mark Shepard. "Subsidizing health insurance for low-income adults: Evidence from Massachusetts." *American Economic Review* 109, no. 4 (2019): 1530-1567

Utility Maximization and Consumer Choice

The axioms of consumer preference theory

The axioms of consumer preference theory were developed for three purposes:

1. Portray rational behavior
2. Mathematical representation of utility functions
3. Derive “well-behaved” demand curves

Utility functions: Cardinal and ordinal

A consumer's utility from consumption of a given bundle A is determined by a personal *utility function*.

Cardinal utility function

- $U(A)$ is a cardinal number: U : consumption $bundle \longrightarrow \mathbb{R}$ measured in "utils"

Ordinal utility function

- U provides a "ranking" or "preference ordering" over bundles.

U:(A,B)$\longrightarrow \begin{cases} A \;^{P}\; B \\ B \;^{P}\; A \\ A \;^{I}\; B \end{cases}$

Cardinal vs. ordinal utility functions

- Problems with **cardinal** utility functions
  1. Difficult to find the appropriate measurement index (metric)
  2. Invite us to make interpersonal comparisons of utility, which is problematic. Want to focus on *intrapersonal* choices
- Using unit-free **ordinal** utility functions avoids these problems
- Surprisingly, we can build a lot of theoretical structure using only ordinal properties of utility functions

Axiom 1: Completeness

Axiom 1: Preferences are complete ("completeness")

- For any two bundles A and B, a consumer can establish a preference ordering.

1. $A \ ^{P} \ B$
2. $B \ ^{P} \ A$
3. $A \ ^{I} \ B$

Axiom 2: Transitivity

Axiom 2: Preferences are transitive ("transitivity")

- For any consumer if $A \ ^P \ B$ and $B \ ^P \ C$ then it must be that $A \ ^P \ C$.
- Consumers are consistent in their preferences

Axiom 3: Completeness, transitivity, and continuity

Axiom 3: Preferences are continuous ("continuity")

- If $A \ ^{P} \ B$ and C lies within an ε radius of B then $A \ ^{P} \ C$.
- We need continuity to derive well-behaved demand curves.

Axioms: Completeness, transitivity, and continuity

- *Axiom 1: Preferences are complete ("completeness")*
- *Axiom 2: Preferences are transitive ("transitivity")*
- *Axiom 3: Preferences are continuous ("continuity")*

Theorem

If Axioms 1–3 are obeyed, then we can define a cardinal utility function that represents the individual's preference.

(Note: this theorem should be interpreted as an "as if" statement. We do not believe that consumers literally have utility functions)

Indifference curves

- The indifference curve $IC(\overline{U})$ is the set of consumption bundles that generate utility level $\overline{U}$ for a utility function U

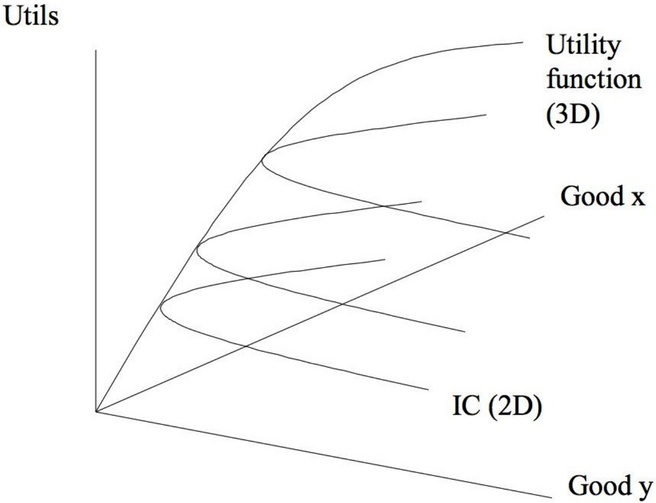
- An *Indifference Curve Map* is a sequence of indifference curves defined over every utility level:

$$\{IC(0), IC(\varepsilon), IC(2\varepsilon), ...\}$$

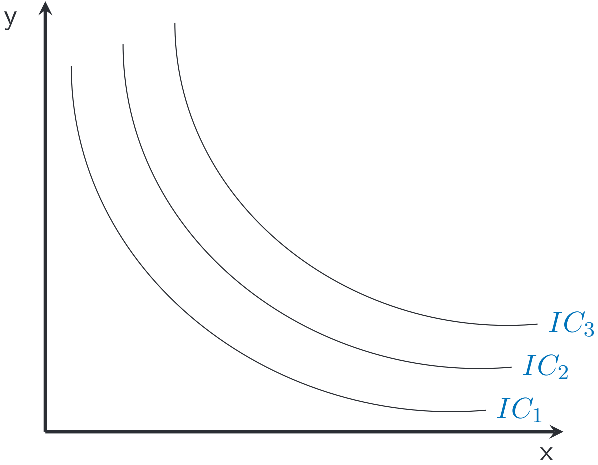
  with a small positive value for ε

3D indifference curve map – Autor will attempt to draw

Indifference curves



Indifference curves



$$\left.\begin{array}{l} IC_3 \longrightarrow \text{ Utility level } U_3 \\ IC_2 \longrightarrow \text{ Utility level } U_2 \\ IC_1 \longrightarrow \text{ Utility level } U_1 \end{array}\right\} U_3 > U_2 > U_1$$

Axiom 4: Non-satiation (never get enough)

We usually use two additional axioms

- Introduced to reflect observed behavior and to simplify
- But, they are not *necessary* for a theory of rational choice

Axiom 4: Non-Satiation

- Given two bundles X and Y, if $X_A = X_B$ and $Y_A > Y_B$ then $A \ ^P \ B$, regardless of the levels of X_A, X_B, Y_A, Y_B

- Implications:
  1. The consumer always places positive value on more consumption
  2. Indifference curve map stretches out endlessly

Axiom 5: Diminishing marginal rate of substitution

- “The more of something you have, the less you value a bit more of it (relative to alternatives)”
  - Captures, what we believe, is a fundamental feature of human preferences
  - Role in consumer theory:
    - Makes the mathematics of consumer theory much simpler
    - Avoids consumers spending all their money on one good
- Need to define **Marginal Rate of Substitution** first

Marginal rate of substitution

Definition (Marginal rate of substitution)

MRS measures willingness to trade one bundle for another

Example

- Bundle $A =$ (1 cup of coldbrew, 12 pieces of sushi)
- Bundle $B =$ (2 cups of coldbrew, 8 pieces of sushi)
- Let's say that the consumer is *indifferent* between these two bundles
- Consumer is indifferent to having 4 fewer pieces of sushi for 1 more cup of coldbrew

$$MRS(\text{cups of coldbrew for sushi}) = |-4|$$

$$MRS\left[\frac{\text{CB}}{\text{Sushi}}\right]_{U=\bar{u}} \equiv \left[\frac{\Delta\text{Sushi}}{\Delta\text{CB}}\right]_{U=\bar{u}}$$

- MRS is measured along an indifference curve and *may* vary along an indifference curve
- MRS is defined relative to some bundle (starting point)

Marginal rate of substitution

– By definition, utility is constant along an indifference curve:

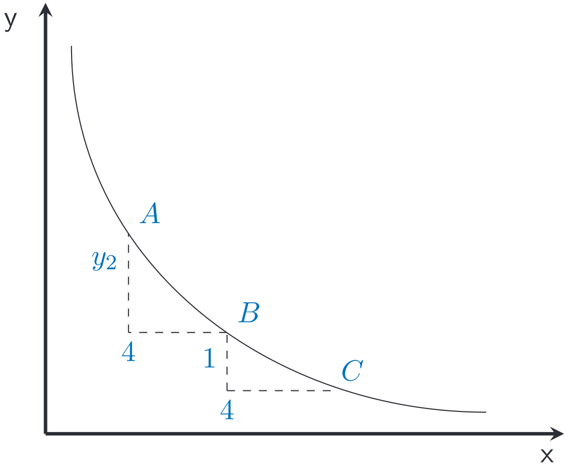
$$\begin{aligned}
\bar{U} &= U(x, y) \\
0 &= \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy \\
0 &= MU_x dx + MU_y dy \\
-\tfrac{dy}{dx} &= \frac{MU_x}{MU_y} = \text{MRS of x for y}
\end{aligned}$$

– MRS of x for y is the marginal utility of x divided by the marginal utility of y (holding total utility constant), which is equal to $-dy/dx.$

– "How much y do you need to compensate for a unit loss in x?"

Marginal rate of substitution

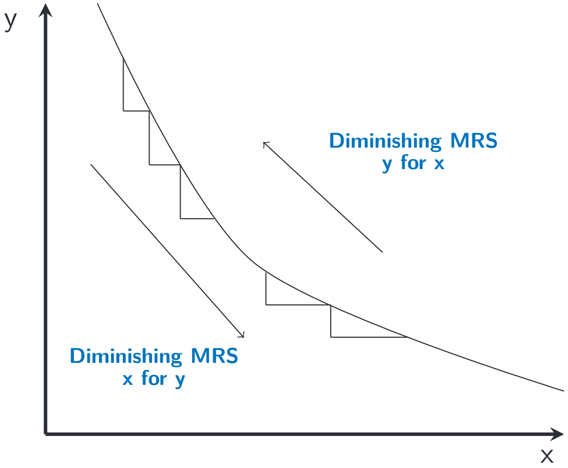
- MRS must always be evaluated at some particular point



Axiom 5: Diminishing marginal rate of substitution

Axiom 5: The MRS of x for y decreases as x increases relative to y

– The ratio MU_x/MU_y is decreasing in x



Convexity and MRS

- Diminishing MRS implies that consumers prefer diversity in consumption
- A convex utility function exhibits diminishing MRS

Definition

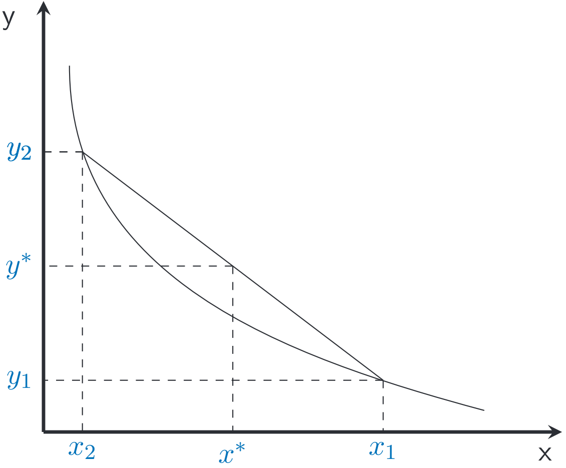
A function $U(x, y)$ is convex if for any arguments (x_1, y_1) and (x_2, y_2) where $(x_1, y_1) \neq (x_2, y_2)$:

$$U(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2) \geq \alpha U(x_1, y_1) + (1-\alpha)U(x_2, y_2),$$

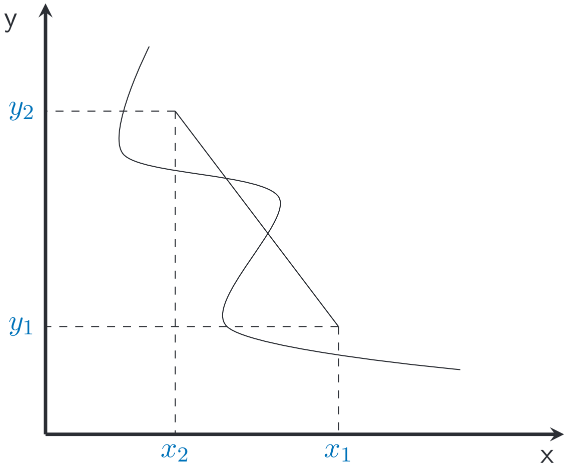
where $\alpha \in (0, 1)$.

Example of convex utility function

A utility function $U(\cdot)$ exhibits diminishing MRS iff the indifference curves of $U(\cdot)$ are convex.

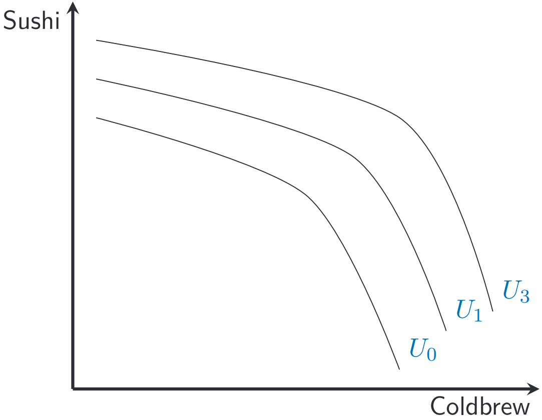


Example of non-convex utility function



Example of concave utility function

- Suppose you love coldbrew and sushi, but dislike consuming them together



- If your indifference curves were concave as above, you should not diversify consumption

Back to Indifference Curves

Properties of indifference curves

1. Every consumption bundle lies on some indifference curve
   (Axiom 1: Completeness)

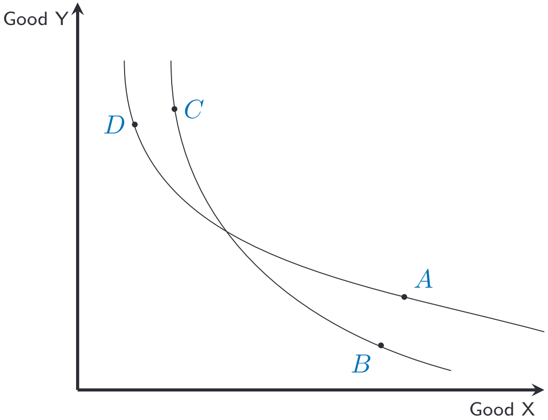
2. Indifference curves are smooth
   (Axiom 3: Continuity)

3. Indifference curves are convex
   (Axiom 5: Diminishing MRS)

4. Indifference curves cannot intersect ...

Non-crossing of indifference curves

- Proof: say two indifference curves intersect:



- According to these indifference curves, (i) $A \ ^P \ B$ (by non-satiation), (ii) $B \ ^I \ C$, (iii) $C \ ^P \ D$ (by non-satiation), (iv) $D \ ^I \ A$
- By transitivity, $A \ ^P \ D$ and $A \ ^I \ D$, which is a contradiction

Cardinal vs ordinal utility

- Utility function $U(x, y) = f(x, y)$ is cardinal
  - It reads off "utils" as a function of consumption
  - But, choices are inherently ordinal
- We'd like to relax the notion of utility functions to make them ordinal rather than cardinal
  - That is, we want 'same preferences' but we don't care about units
- What does it mean for two utility functions to have the "**same preferences**"?

Monotone transformation

Q: How do we preserve properties of utility that we care about and believe in without imposing cardinal properties?

We say that a utility function is defined only up to a **positive monotone transformation**

Definition (Monotone Transformation)

Let I be an interval on the real line $(\mathbb{R})$ then: $g : I \longrightarrow \mathbb{R}$ is a monotone transformation if g is a strictly increasing function on I.

If $g(\cdot)$ if a positive monotone transformation of $f(\cdot)$, we will say they $g(\cdot)$ and $f(\cdot)$ are identical for purposes of utility theory

If $g(x)$ is differentiable and monotone, then $g'(x) > 0 \; \forall x$

Monotone and non-monotone transformations

Examples

Let y be defined on $\mathbb{R}$, and let x be defined as:

1. $x = 2y + 1$ YES
2. $x = \exp(y)$ YES
3. $x = abs(y)$ NO
4. $x = y^3$ YES
5. $x = -\frac{1}{y}$ YES
6. $x = \max(y^2, y^3)$ NO
7. $x = 2y - y^2$ NO

Monotone Transformation of a Utility Function

- If $U_2(.)$ is a monotone transformation of $U_1(.)$, then:
    1. $U_2(.) = f(U_1(.))$ where $f(.)$ is monotone in U_1
    2. U_1 and U_2 exhibit identical preference rankings
    3. MRS of $U_1(\overline{U})$ and $U_2(\overline{U})$ are the same
    4. U_1 and U_2 are equivalent for consumer theory

- Example: $U(x, y) = x^{\alpha} y^{\beta}$ (Cobb-Douglas)

Cobb-Douglas example: Monotone transformation

- What is the MRS along an indifference curve U_0?

$$\begin{aligned} U_0 &= x_0^{\alpha} y_0^{\beta} \\ dU_0 &= \alpha x_0^{\alpha-1} y_0^{\beta} dx + \beta x_0^{\alpha} y_0^{\beta-1} dy \\ \left.\frac{dy}{dx}\right|_{U=U_0} &= -\frac{\alpha x_0^{\alpha-1} y_0^{\beta}}{\beta x_0^{\alpha} y_0^{\beta-1}} = \frac{\alpha}{\beta}\frac{y_0}{x_0} = \frac{\partial U/\partial x}{\partial U/\partial y} \end{aligned}$$

- Consider now a monotonic transformation of U:

$$\begin{aligned} U^1(x,y) &= x^{\alpha} y^{\beta} \\ U^2(x,y) &= \ln(U^1(x,y)) \\ U^2 &= \alpha \ln x + \beta \ln y \end{aligned}$$

Cobb-Douglas example: Monotone transformation

- What is the MRS of U^2 along an indifference curve?

$$\begin{aligned} U_0^2 &= \ln U_0 = \alpha \ln x_0 + \beta \ln y_0 \\ dU_0^2 &= \frac{\alpha}{x_0}dx + \frac{\beta}{y_0}dy = 0 \\ -\left.\frac{dy}{dx}\right|_{U^2=U_0^2} &= \frac{\alpha}{\beta}\frac{y_0}{x_0} = \frac{\partial U/\partial x}{\partial U/\partial y} \end{aligned}$$

- which is the same as we derived for U^1
- This is actually just an application of the chain rule

Why monotone transformations preserve the MRS

How we know that monotonic transformations always preserve the MRS of a utility function?

- Let $U = f(x, y)$ be a utility function
- Let $g(U)$ be a monotonic transformation of $U = f(x, y)$ and differentiable
- The MRS of $g(U)$ along an indifference curve where $U_0 = f(x_0, y_0)$ and $g(U_0) = g(f(x_0, y_0))$
- By totally differentiating this equality we can obtain the MRS.

$$dg(U_0) = g'(f(x_0, y_0))f_x(x_0, y_0)dx + g'(f(x_0, y_0))f_y(x_0, y_0)dy$$

$$-\frac{dy}{dx}\bigg|_{g(U)=g(U_0)} = \frac{g'(f(x_0, y_0))f_x(x_0, y_0)}{g'(f(x_0, y_0))f_y(x_0, y_0)} = \frac{f_x(x_0, y_0)}{f_y(x_0, y_0)} = \frac{\partial U/\partial x}{\partial U/\partial y}$$

which is the MRS of the original function $U(x, y)$

Utility maximization

Utility maximization subject to budget constraint

- Maximize utility subject to budget constraint
  - Utility function (preferences)
    - Consumer wants to maximize $U(x)$
  - Budget constraint
    - Has a maximum $ amount that he/she can spend
  - Price vector
    - Total possible purchases depend on price of goods
  - We know that the consumer can make a choice because of completeness

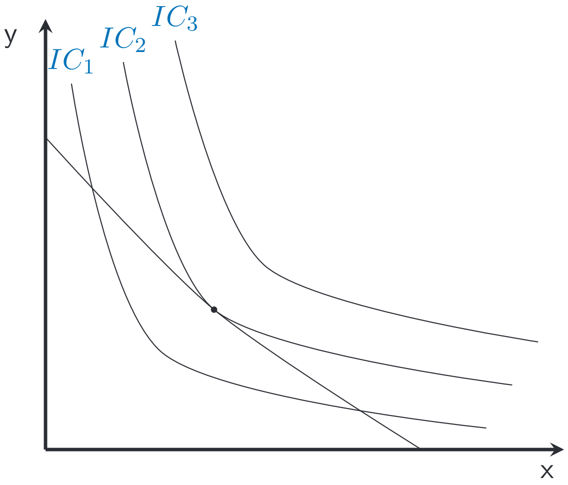
- Characteristics of the solution
  - Budget exhaustion (non-satiation)
  - For most solutions: psychic trade-off = monetary payoff
    - Psychic trade-off is MRS
    - Monetary trade-off is the price ratio

Consumer's problem

- Utility maximization corresponds to point A
  - The slope of the budget set is equal to $-\frac{p_x}{p_y}$
  - The slope of each indifference curves is given by the MRS
- Every bundle is on some indifference curve (completeness)

Interior solutions

- Two types of solutions: Interior solutions and corner solutions

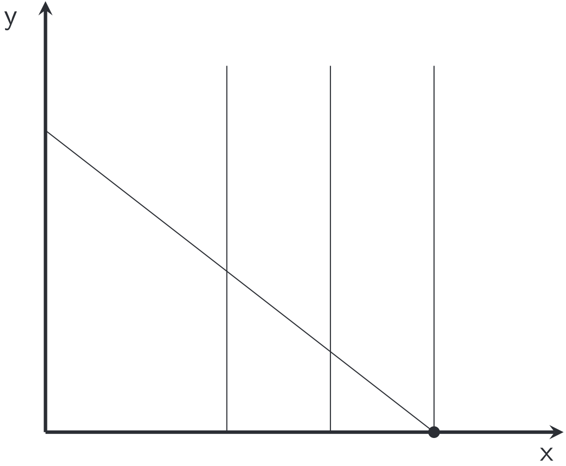


Oddball cases —

We won't be focusing on these

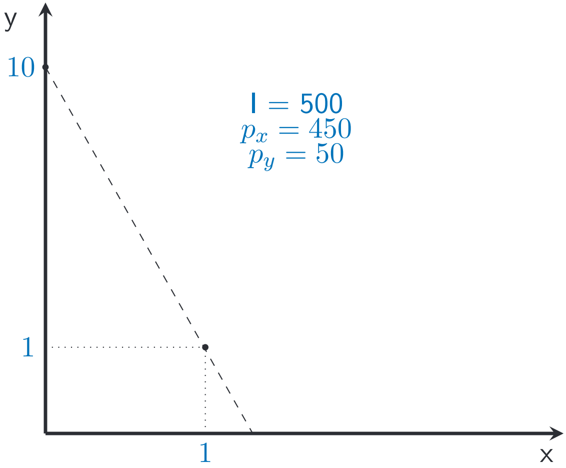
Corner solutions

- Vertical indifference curves implies that the consumer is indifferent to the consumption of good y



Integer constraints

- Another type of “corner” solution can result from indivisibilities/integer constraints
  - Only two bundles are feasible



Non-negativity constraints — Autor will attempt to draw

Utility maximization

The math you've been waiting for!

Mathematical solution to the Consumer's Problem

- Problem:

$$\max_{x,y} U(x,y)$$
$$s.t. \;\; p_x x + p_y y \leq I$$

- Lagrangian version:

$$\max_{x,y,\lambda} \; L(x,y,\lambda) \;\; = \;\; \max_{x,y,\lambda} \; U(x,y) + \lambda(I - p_x x - p_y y)$$

- First-order conditions (FOC):

$$1. \quad \frac{\partial L}{\partial x} = U_x - \lambda p_x = 0$$
$$2. \quad \frac{\partial L}{\partial y} = U_y - \lambda p_y = 0$$
$$3. \quad \frac{\partial L}{\partial \lambda} = I - p_x x - p_y y = 0$$

Mathematical solution to the Consumer's Problem

$$
\begin{aligned}
1.\quad & \frac{\partial L}{\partial x} &=& \; U_x - \lambda p_x = 0 \\
2.\quad & \frac{\partial L}{\partial y} &=& \; U_y - \lambda p_y = 0 \\
3.\quad & \frac{\partial L}{\partial \lambda} &=& \; I - p_x x - p_y y = 0
\end{aligned}
$$

- Equation (3) states that the consumer spends all of her money ('budget exhaustion')
- Rearranging (1) and (2):

$$\frac{U_x}{U_y} = \frac{p_x}{p_y}$$

  - Q: What does this expression mean *intuitively*?
  - *The psychic trade-off is equal to the monetary trade-off between the two goods*

Mathematical solution to the Consumer's Problem

$$
\begin{aligned}
1. \quad \frac{\partial L}{\partial x} &= U_x - \lambda p_x = 0 \\
2. \quad \frac{\partial L}{\partial y} &= U_y - \lambda p_y = 0 \\
3. \quad \frac{\partial L}{\partial \lambda} &= I - p_x x - p_y y = 0
\end{aligned}
$$

- Notice that:

$$
\begin{aligned}
\frac{U_x}{p_x} &= \lambda \\
\frac{U_y}{p_y} &= \lambda
\end{aligned}
$$

- What is the meaning of λ?

An example problem

Utility maximization: An example problem

- Consider the following example problem:

$$U(x, y) = \frac{1}{4} \ln x + \frac{3}{4} \ln y$$

- Notice that this utility function satisfies all axioms:
  1. Completeness, transitivity, continuity
  2. Non-satiation: $U_x = \frac{1}{4x} > 0$ for all $x > 0$. $U_y = \frac{3}{4y} > 0$ for all $y > 0$
  3. Diminishing marginal rate of substitution:
     - Along an indifference curve: $\bar{U} = \frac{1}{4} \ln x_0 + \frac{3}{4} \ln y_0$.
     - Totally differentiate: $0 = \frac{1}{4x_0} dx + \frac{3}{4y_0} dy$.
     - Yields marginal rate of substitution $-\frac{dy}{dx}|_{\bar{U}} = \frac{U_x}{U_y} = \frac{4y_0}{12x_0}$.

Utility maximization: An example problem

– Example values: $p_x = 1,\ p_y = 2,\ I = 12$. Write the Lagrangian for this utility function given prices and income:

$$\begin{aligned} & \max_{x,y} U(x,y) \\ s.t.\ \ p_x x + p_y y &\leq I \\ L &= \frac{1}{4}\ln x + \frac{3}{4}\ln y + \lambda(12 - x - 2y) \end{aligned}$$

$$\begin{aligned} 1. \quad \frac{\partial L}{\partial x} &= \frac{1}{4x} - \lambda = 0 \\ 2. \quad \frac{\partial L}{\partial y} &= \frac{3}{4y} - 2\lambda = 0 \\ 3. \quad \frac{\partial L}{\partial \lambda} &= 12 - x - 2y = 0 \end{aligned}$$

Utility maximization: An example Problem

$$
\begin{aligned}
L &= \frac{1}{4}\ln x + \frac{3}{4}\ln y + \lambda(12 - x - 2y) \\
1. \quad \frac{\partial L}{\partial x} &= \frac{1}{4x} - \lambda = 0 \\
2. \quad \frac{\partial L}{\partial y} &= \frac{3}{4y} - 2\lambda = 0 \\
3. \quad \frac{\partial L}{\partial \lambda} &= 12 - x - 2y = 0
\end{aligned}
$$

– Rearranging (1) and (2), we have

$$
\begin{aligned}
\frac{2/4x}{3/4y} &= \frac{2}{3} \\
\frac{U_x}{U_y} &= \frac{p_x}{p_y}
\end{aligned}
$$

An Example Problem

- We have:

$$\lambda = \frac{1}{4x^*} = \frac{3}{8y^*} \Rightarrow x^* = \frac{2}{3}y^*.$$

- Now using the budget constraint:

$$x + 2y = 12 \Rightarrow \frac{2}{3}y^* + 2y^* = \frac{8}{3}y^* = 12$$

- And therefore: $y^* = 4.5$, $x^* = 3$

Interpreting the Lagrange multiplier (λ) — The mathematics of ‘bang for the buck’

Interpretation of λ, the Lagrange multiplier

- At the solution, the following conditions will hold:

$$\frac{\partial U/\partial x_1}{p_1} = \frac{\partial U/\partial x_2}{p_2} = \ldots = \frac{\partial U/\partial x_n}{p_n} = \lambda$$

- So what is $\frac{dU^*}{dI}$? Return to Lagrangian:

$$L = U(x, y) + \lambda(I - p_x x - p_y y)$$

$$\frac{dL}{dI} = \left(U_x \frac{\partial x^*}{\partial I} - \lambda p_x \frac{\partial x^*}{\partial I}\right) + \left(U_y \frac{\partial y^*}{\partial I} - \lambda p_y \frac{\partial y^*}{\partial I}\right) + \lambda$$

- Recall that

$$\frac{U_x}{p_x} = \frac{U_y}{p_y} = \lambda$$

- Substituting for λ at optimal choices, we get that

$$\lambda = \frac{dL}{dI} = \frac{\partial L}{\partial I}$$

- This is the **envelope theorem** at work!

Interpretation of λ, the Lagrange multiplier

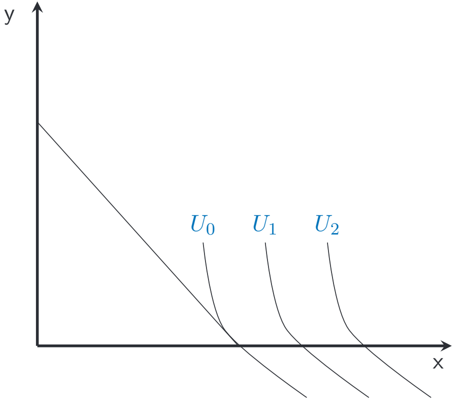
- λ equals the "shadow price" of the budget constraint
  - Additional utils that could be obtained with the next dollar of consumption
  - Note that this shadow price is not uniquely defined since it corresponds to the marginal utility of income in "utils"

- We could have concluded that $dL/dI = \lambda$ directly using the envelope theorem:

$$\frac{dU^*}{dI} = \frac{\partial L}{\partial I} = \lambda$$

Corner solutions and the Lagrangian

- When at a corner solution, a point of tangency need not exist



- Need to modify Lagrangian to include "non-negativity constraints":

$$x \geq 0, y \geq 0$$

The road ahead — in consumer theory

The road ahead – In consumer theory