# 14.03/003 Micro Theory & Public Policy, Fall 2025

## Lecture 5. Consumer Theory and the Carte Blanche Principle

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# The indirect utility function

# Indirect utility function

- For any
  - Budget constraint
  - Utility function
  - Set of prices
- We obtain a set of optimally chosen quantities:

$$\begin{aligned} x_1^* &= x_1(p_1, p_2, ..., p_n, I) \\ &... \\ x_n^* &= x_n(p_1, p_2, ..., p_n, I) \end{aligned}$$

- These quantities solve the problem

$$\max U(x_1, ..., x_n) \; s.t. \; PX \leq I$$

and yield (indirect) utility

$$U(x_1^*(p_1, ..., p_n, I), ..., x_n^*(p_1, ..., p_n, I)) \equiv V(p_1, ..., p_n, I).$$

# Indirect utility function

- The "Indirect utility function", $V(\cdot)$, is the value of maximized utility under given prices and income

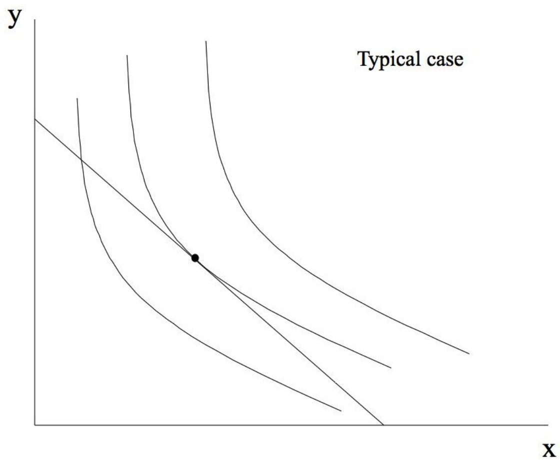
- **Remember the distinction:**
  - **Direct utility**: utility from consumption of $(x_1, ..., x_n)$
  - **Indirect utility**: utility obtained when facing $(p_1, ..., p_n, I)$

# Indirect utility function

*Graphical interpretation*

# Indirect utility function

*Graphical interpretation*



# Indirect utility function

*Example*

$$
\begin{aligned}
\max\ U(x,y) &= x^{0.5}y^{0.5} \\
s.t.\ p_x x + p_y y &\leq I \\
L &= x^{0.5}y^{0.5} + \lambda(I - p_x x - p_y y) \\
\frac{\partial L}{\partial x} &= 0.5x^{-0.5}y^{0.5} - \lambda p_x = 0 \\
\frac{\partial L}{\partial y} &= 0.5x^{0.5}y^{-0.5} - \lambda p_y = 0 \\
\frac{\partial L}{\partial \lambda} &= I - p_x x - p_y y = 0
\end{aligned}
$$

- We obtain the following:

$$\lambda = \frac{0.5x^{-0.5}y^{0.5}}{p_x} = \frac{0.5x^{0.5}y^{-0.5}}{p_y},$$

which simplifies to:

$$x = \frac{p_y y}{p_x}.$$

# Indirect utility function

- Substituting into the budget constraint gives us

$$\begin{aligned} I - p_x \frac{p_y y}{p_x} - p_y y &= 0 \\ p_y y &= \frac{1}{2} I, \quad p_x x = \frac{1}{2} I \end{aligned}$$

$$x^* = \frac{I}{2p_x}, \quad y^* = \frac{I}{2p_y}$$

  - Half of the budget goes to each good

- Thus, a consumer with $U(x, y) = x^{0.5} y^{0.5}$, budget $I$, and facing prices $p_x$ and $p_y$ will choose $x^*$ and $y^*$ and obtain utility:

$$U(x^*, y^*) = \left(\frac{I}{2p_x}\right)^{0.5} \left(\frac{I}{2p_y}\right)^{0.5}.$$

# Indirect utility function

- Thus, the indirect utility for this consumer is

$$V(p_x, p_y, I) = U(x^*(p_x, p_y, I), y^*(p_x, p_y, I)) = \left(\frac{I}{2p_x}\right)^{0.5} \left(\frac{I}{2p_y}\right)^{0.5}$$

- Why bother calculating the indirect utility function?
  - Instead of recalculating the utility level for every set of prices and budget constraints, we can plug in prices and income to get consumer utility
  - Much easier to work with indirect utility f'n (i.e., the maximized utility f'n) than direct utility f'n that needs to be re-maximized in every calculation
  - Will be useful later for analyzing demand and consumer well-being

# Individual (i.e., personal) demand curves

## ‘Marshallian demand’ — Demand as f’n of prices and income

$$d_{x1}\left(p_1, p_2, ..., p_n, I\right)$$

# Individual demand

- Now, let's use the indirect utility function to get demand functions
- Up to now, we have been solving for:
  - Utility as a function of prices and budget
- Implicitly we have already found demand schedules—a demand schedule is immediately implied by an individual utility function
- For any utility function, we can solve for the quantity demanded of each good as a function of its price, holding the price of all other goods constant *and* holding income constant.
- (Next, we'll hold utility constant instead of income)

# Uncompensated (Marshallian) demand

In our previous example where:

$$U(x, y) = x^{0.5} y^{0.5}$$

we derived:

$$\begin{aligned} x(p_x, p_y, I) &= 0.5 \frac{I}{p_x} \\ y(p_x, p_y, I) &= 0.5 \frac{I}{p_y} \end{aligned}$$

In general we will write these demand functions (for individuals) as:

$$\begin{aligned} x_1^* &= d_1(p_1, p_2, ..., p_n, I) \\ x_2^* &= d_2(p_1, p_2, ..., p_n, I) \\ & ... \\ x_n^* &= d_n(p_1, p_2, ..., p_n, I) \end{aligned}$$

# Marshallian (uncompensated) demand

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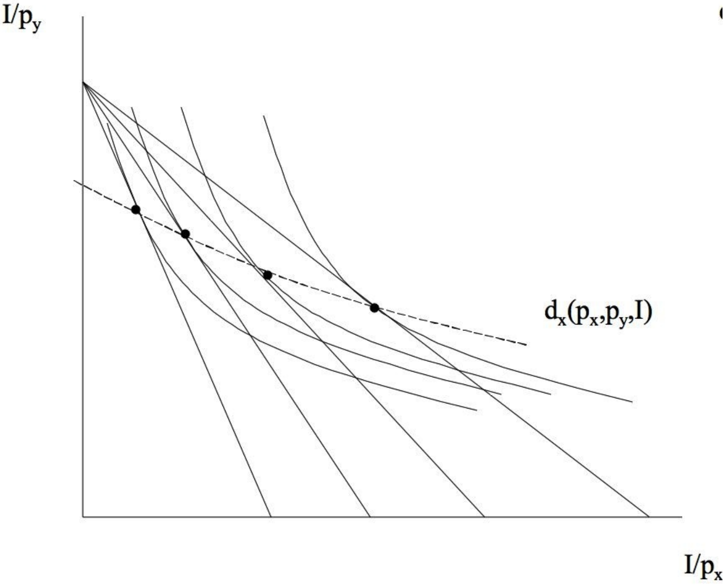

# Income and substitution effects

## (Normal and Inferior goods)

# What happens to demand for a good when its price increases but income is held constant?

- Formally, what is $\partial d_x(p_x, p_y, I)/\partial p_x$.
- Two effects:
  1. It shifts the budget set inward toward the origin for the good whose price has risen. This component is the 'income effect.'
  2. It changes the slope of the budget set so that the consumer faces a different set of market trade-offs. This component is the 'substitution effect.'

# Effect of a price increase on the budget set

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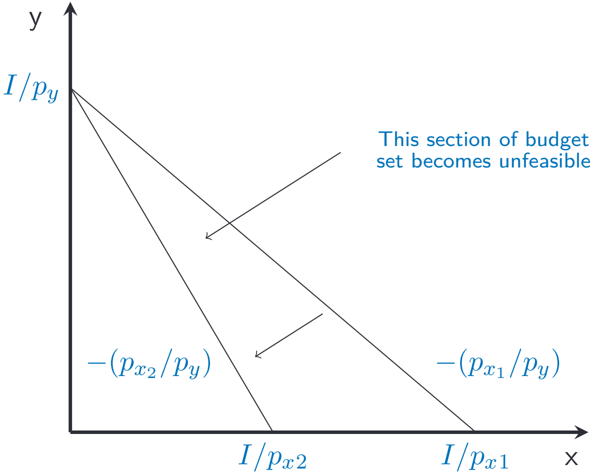


# Substitution effect

- What happens to consumption of $X$ if

$$\frac{p_x}{p_y} \uparrow$$

  *while utility is held constant?*

- Provided that the axiom of diminishing MRS applies, we'll have

$$\frac{\partial h_x(p_x, p_y, U)}{\partial p_x} < 0$$

- Holding utility constant, the substitution effect is *always* negative.

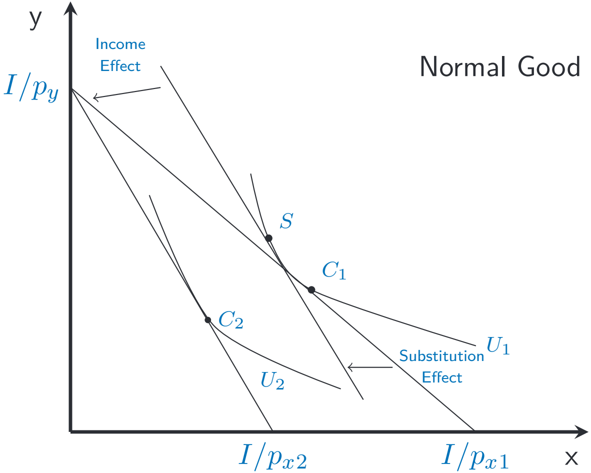
# Income effect

- Defined as

$$\partial d_x(p_x, p_y, I)/\partial I$$

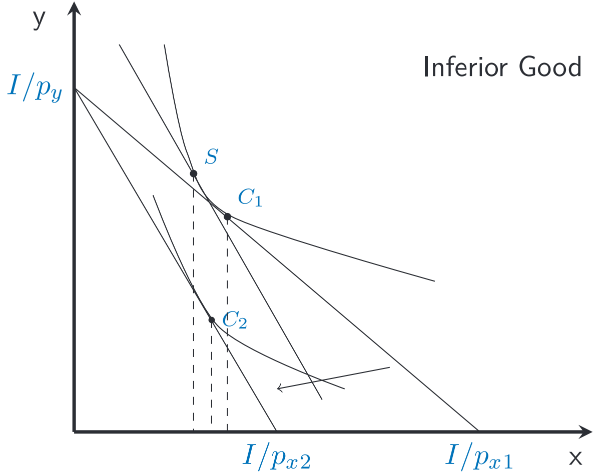
- Can be either negative or positive.
  - If positive, good $X$ is said to be a “normal” good.
  - If negative, good $X$ is said to be an “inferior” good.
  - Inferior goods can be further subdivided in “weakly” and “strongly” inferior goods:
    - We'll come back to this point soon

# Income and substitution effects: Normal good



y
Income
Effect
Normal Good
$I/p_y$
$S$
$C_1$
$C_2$
$U_1$
Substitution
Effect
$U_2$
$I/p_{x2}$
$I/p_{x1}$
x

# Income and substitution effects: Inferior good



y
Inferior Good
$I/p_y$
$S$
$C_1$
$C_2$
$I/p_{x2}$
$I/p_{x1}$
x

# Normal and Inferior goods

*Summary*

- For a normal good ($\frac{\partial d_x}{\partial I} > 0$), the income and substitution effects are complementary.
- For an inferior good ($\frac{\partial d_x}{\partial I} < 0$), the income and substitution effects are countervailing.
- For a Giffen good (AKA, *strongly inferior, abnormal*), the income effect dominates: $\left|\frac{\partial d_x}{\partial I} \cdot X\right| > \left|\frac{\partial h_x}{\partial p_x}\right|$. Note **both** are negative. (*We'll cover this soon—not today*)

# Compensated (Hicksian) demand

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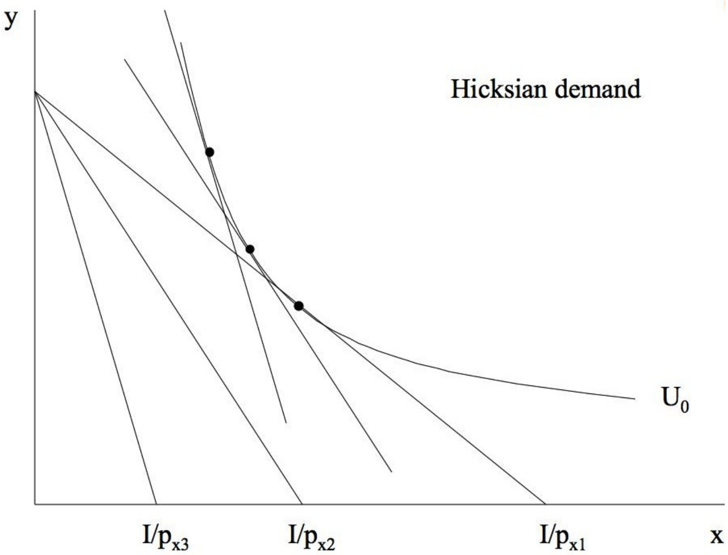


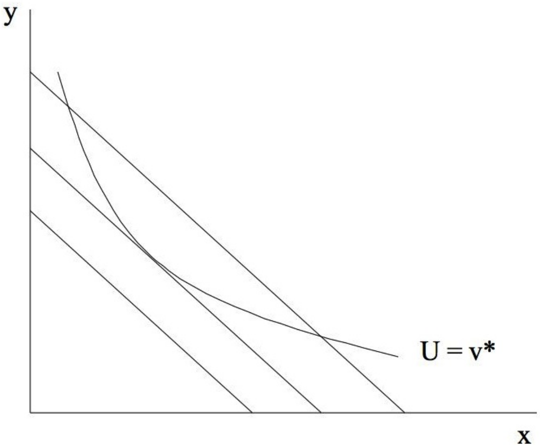
# The Expenditure Function

# Expenditure function

## *Graphical interpretation*

# Expenditure function

*Graphical interpretation*



# Expenditure function

*Example*

$$\begin{aligned} \min\ E &= p_x x + p_y y \\ s.t.\ x^{0.5} y^{0.5} &\geq U_p \end{aligned}$$

$$L = p_x x + p_y y + \lambda \left( U_p - x^{0.5} y^{0.5} \right)$$

$$\begin{aligned} \frac{\partial L}{\partial x} &= p_x - \lambda 0.5 x^{-0.5} y^{0.5} = 0 \\ \frac{\partial L}{\partial y} &= p_y - \lambda 0.5 x^{0.5} y^{-0.5} = 0 \\ \frac{\partial L}{\partial \lambda} &= U_p - x^{0.5} y^{0.5} = 0 \end{aligned}$$

# Expenditure function

*Example continued*

The first two of these equations simplify to:

$$x = yp_y/p_x$$

We substitute into the constraint $U_p = x^{0.5}y^{0.5}$ to get

$$U_p = \left(\frac{p_y y}{p_x}\right)^{0.5} y^{0.5}$$

$$x^* = \left(\frac{p_y}{p_x}\right)^{0.5} U_p, \; y^* = \left(\frac{p_x}{p_y}\right)^{0.5} U_p$$

These are our Hicksian ('compensated') demand functions

$$h_x\left(p_x, p_y, U_p\right) = \left(\frac{p_y}{p_x}\right)^{0.5} U_p \text{ and } h_y\left(p_x, p_y, U_p\right) = \left(\frac{p_x}{p_y}\right)^{0.5} U_p$$

# Expenditure function

*Example continued*

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$$h_x\left(p_x, p_y, U_p\right) = \left(\frac{p_y}{p_x}\right)^{0.5} U_p \text{ and } h_y\left(p_x, p_y, U_p\right) = \left(\frac{p_x}{p_y}\right)^{0.5} U_p$$

Now calculate expenditure substituting $h_x, h_y$ into the constraint $U_p = x^{0.5}y^{0.5}$

$$E^* = p_x \left(\frac{p_y}{p_x}\right)^{0.5} U_p + p_y \left(\frac{p_x}{p_y}\right)^{0.5} U_p$$

$$= 2p_x^{0.5}p_y^{0.5}U_p$$

# Expenditure function

*What is it good for?*

- Expenditure function answers the question: "How much do we need to compensate consumer (pos or negative) for a change in prices or policy to keep them on the same indifference curve?"
- We don't know what "utils" are, but can observe what people are willing to pay, or give up, to obtain specific things
- Allows 'monetizing' otherwise incommensurate trade-offs to evaluate costs and benefits
- Essential tool for public policy analysis
  - We are not interested in money as a measure of utility
  - We are interested in trade-offs someone would make based on their preferences
  - Money metric enables those trade-offs to be quantified

# Expenditure Function $\leftrightarrow$ Indirect Utility function

# Expenditure function $\leftrightarrow$ Indirect utility function

$$
\begin{aligned}
V(p_x, p_y, I_0) &= U_0 \\
E(p_x, p_y, U_0) &= I_0 \\
V(p_x, p_y, E(p_x, p_y, U_0)) &= U_0 \\
E(p_x, p_y, V(p_x, p_y, I_0)) &= I_0
\end{aligned}
$$

# The indirect utility function and expenditure function are inverses

*Back to Cobb-Douglas example*

The dual problem gave us expenditures (budget requirement) as a function of utility and prices.

$$x_p^* = \frac{I}{2p_x},\ y_p^* = \frac{I}{2p_y},\ U^* = \left(\frac{I}{2p_x}\right)^{0.5}\left(\frac{I}{2p_y}\right)^{0.5}$$

Now plug these into expenditure function:

$$E^* = 2U_p p_x^{0.5} p_y^{0.5} = 2\left(\frac{I}{2p_x}\right)^{0.5}\left(\frac{I}{2p_y}\right)^{0.5} p_x^{0.5} p_y^{0.5} = I$$

# The Carte Blanche Principle

# The Carte Blanche Principle

- Implication of consumer theory – consumers make optimal choices given
  - Prices, constraints, and income.

- Carte Blanche principle:
  - Consumers are always weakly better off receiving a cash transfer than an in-kind transfer of identical monetary value
  - Why?

# In-kind transfers

- Examples of in-kind transfers given to U.S. citizens:
  - Food Stamps, housing vouchers, health insurance, subsidized educational loans, child care services, job training, etc.

- Economic theory suggests
  - Relative to equivalent cash transfer, in-kind transfers *constrains* consumer choice
  - If consumers are rational, constraints on choice cannot be beneficial

- Consider a consumer with income $I = 2,000/\text{mo}$ choosing between necessities (food, housing, transportation, clothing, etc.) and health insurance at normalized prices

$$p_N = 1, p_H = 1$$

per unit:

$$\max_{N,H} U(N,H)$$
$$s.t.\ \ N + H \leq 2,000$$

# In-kind transfers

- The government decides to provide a health insurance subsidy of $\$400$/mo
  - Consumer can now spend up to $\$2,400$ on health insurance but no more than $\$2,000$ on necessities
- The consumer's problem is:

$$\max_{N,H} U(N,H)$$
$$s.t.\ \ N + H \leq 2,400$$
$$H \geq 400$$

- Alternatively, if the government had provided $400$ dollars in cash, the problem would be:

$$\max_{N,H} U(N,H)$$
$$s.t.\ \ N + H \leq 2,400.$$

# In-kind transfers

- The government's transfer therefore has two components:
  1. An expansion of the budget set from $I = 2,000$ to $I' = I + 400$
  2. The imposition of the constraint that $H \geq 400$.

- The canonical economist's question is:
  - Why relax one constraint and impose another, when you could simply relax the constraint?
  - Government's cost is the same in either case, but consumer can only be harmed by the new constraint?

# Methodological interlude:

# Using discontinuities to learn about causal effects

# Using discontinuities to learn about causal effects

- Arbitrary cutoffs are necessary for administration
- Why are they useful for researchers?
- Define a variable $X$ that is used to determine whether a person (or unit) $i$ is or is not assigned to treatment, depending on if they are above or below the cutoff.
  - $X$ could be the percentage of voters for candidate A
  - $X$ could be the exact hour/minute/second of birth.
- We will refer to $X$ as the *running variable*, and we'd like that variable to be continuous

# Using discontinuities to learn about causal effects

- Imagine there are two underlying relationships between potential outcomes and treatment, represented by $E[Y_{i1}|X_i]$ and $E[Y_{i0}|X_i]$

- Thus at each value of $X_i$, the causal effect of treatment is

  $E\left[T|X_i = x\right] = E[Y_{i1}|X_i = x] - E[Y_{i0}|X_i = x]$

- Let's say that individuals to the right of a cutoff $c$ (e.g., $X_i \geq 0.5$) are exposed to treatment, while those to the left $(X_i < 0.5)$ are denied treatment

- We therefore observe $E\left[Y_{i1}|X_i\right]$ to the right of the cutoff and $E\left[Y_{i0}|X_i\right]$ to the left of the cutoff

# Using discontinuities to learn about causal effects

# Relationship b/w GPA in Econ 1+2 and Econ major at UCSC in 2008-12

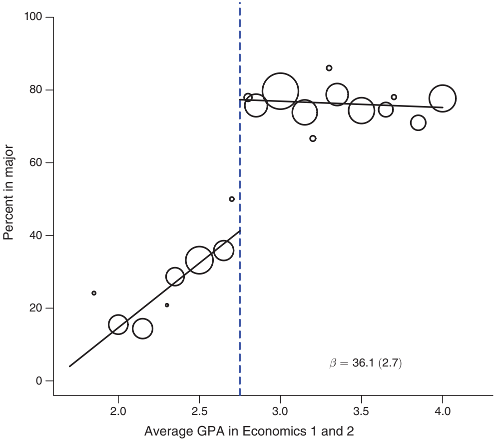


Figure 1. The Effect of the UCSC Economics GPA Threshold on Majoring in Economics

*Notes:* Each circle represents the percent of economics majors ($y$-axis) among 2008–2012 UCSC students who earned a given *EGPA* in Economics 1 and 2 ($x$-axis). The size of each circle corresponds to the proportion of students who earned that *EGPA*. *EGPA*s below 1.8 are omitted, leaving 2,839 students in the sample. Fit lines and beta estimate (at the 2.8 GPA threshold) from linear RD specification; standard error (clustered by *EGPA*) in parentheses.


Bleemer & Mehta,2022

# Using discontinuities to learn about causal effects

- Consider units $i$ that are arbitrarily close (within $\varepsilon$) to threshold. Plausibly:

$$\lim_{\varepsilon \downarrow 0} E\left[Y_{i1} | X_i = c + \varepsilon\right] = \lim_{\varepsilon \uparrow 0} E\left[Y_{i1} | X_i = c + \varepsilon\right],$$
$$\lim_{\varepsilon \downarrow 0} E\left[Y_{i0} | X_i = c + \varepsilon\right] = \lim_{\varepsilon \uparrow 0} E\left[Y_{i0} | X_i = c + \varepsilon\right].$$

- That is, for units that are *almost identical*, we may be willing to assume that had both been treated (or not treated), their outcomes would have been arbitrarily similar

- If this assumption is plausible, we can form a **Regression Discontinuity** estimate of the causal effect of treatment on outcome $Y$ using the contrast:

$$\hat{T} = \lim_{\varepsilon \downarrow 0} E\left[Y_i | X_i = c + \varepsilon\right] - \lim_{\varepsilon \uparrow 0} E\left[Y_i | X_i = c + \varepsilon\right],$$

which in the limit is equal to:

$$T = E\left[Y_{i1} - Y_{i0} | X_i = c\right]$$

*Tada! Our regression discontinuity estimator*

# GPA in Econ 1+2 in '07/08 v. annual earnings '17/18, UCSC ugrads

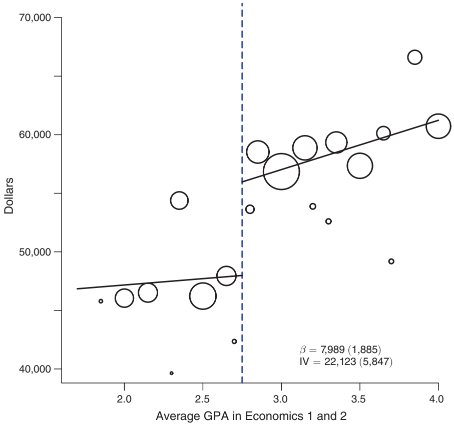


FIGURE 2. THE EFFECT OF THE UCSC ECONOMICS GPA THRESHOLD ON ANNUAL WAGES

*Notes:* Each circle represents the mean 2017–2018 wages ($y$-axis) among 2008–2012 UCSC students who earned a given *EGPA* in Economics 1 and 2 ($x$-axis). The size of each circle corresponds to the proportion of students who earned that *EGPA*. 2017–2018 wages are the mean EDD-covered California wages in those years, omitting zeroes. Wages are CPI adjusted to 2018 and winsorized at 2 percent above and below. *EGPA*s below 1.8 are omitted, leaving 2,446 students with observed wages. Fit lines and beta estimate (at the 2.8 GPA threshold) from linear RD specification and instrumental variable specification (with majoring in economics as the endogenous variable); standard errors (clustered by *EGPA*) in parentheses.

# Relationship with other academic outcomes

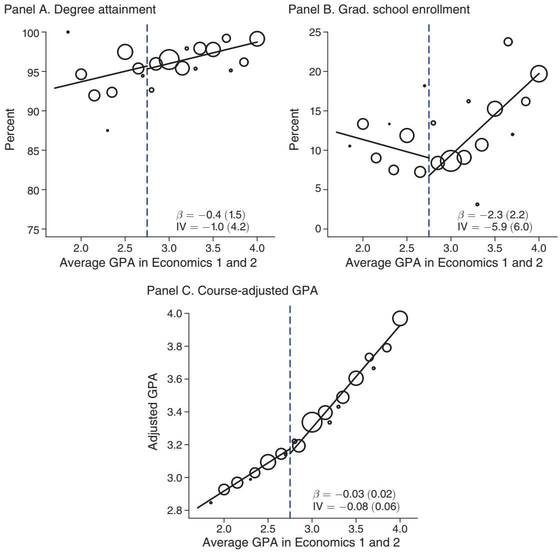


FIGURE 4. THE EFFECT OF ECONOMICS MAJOR ACCESS ON EDUCATION AND ATTAINMENT



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# Relationship with post-college outcomes

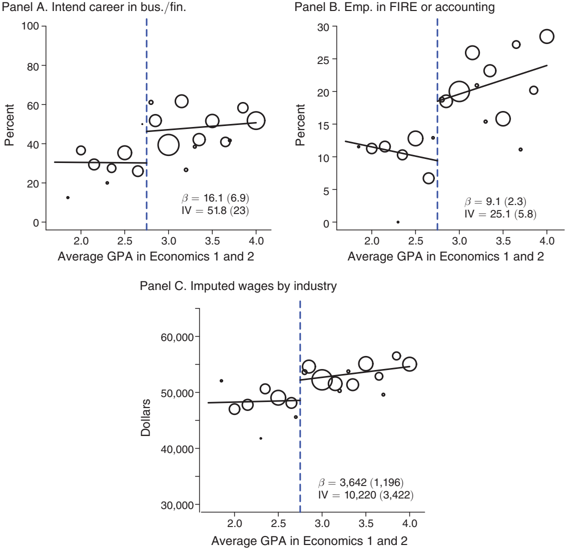


Figure 5. Effect of Economics Major Access on Industry Preferences and Employment



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