1 Overview of "Lemons" Market Unraveling

Analysis of markets with asymmetric information can be intimidating, but a lot of the intuition from standard markets carries through. The main distinction is that each side cares only about the good or service they buy/sell in standard markets, not the identity of the buyer or selling with whom they transact. This changes with asymmetric information, since one side can't actually observe the type of good or service they're buying/selling. Therefore they need to use information about the buyer/seller to infer the quality of the good or service.

Market unraveling broadly describes when mutually advantageous trades are unable to be realized due to asymmetric information. In the market for lemons, the setup is that sellers know quality while buyers do not.

Key steps to understand how markets unravel: Before listing for sale, what information does a seller have? For a given item listed for sale, what information does a buyer have? How does a seller take then take this into account?

Let's go through a mathematical example.

- Quality of good θ uniformly distributed on $[0, \bar{\theta}]$ observed only by seller
- Buyer utility $\theta - p$
- Seller utility $p - r(\theta)$
- Competitive equilibrium is single price (why?) p^* s.t. all sellers value good less than p^* and buyers expect to receive p^* quality good

Formal equilibrium conditions:

$$\Theta^* = \{\theta : r(\theta) \leq p^*\} \text{(i.e. types of goods sold in equilibrium)} \quad (1)$$
$$p^* = E[\theta | \theta \in \Theta^* \text{(i.e. equilibrium price given types of goods sold in equilibrium)}] \quad (2)$$

1.1 Graphical analysis

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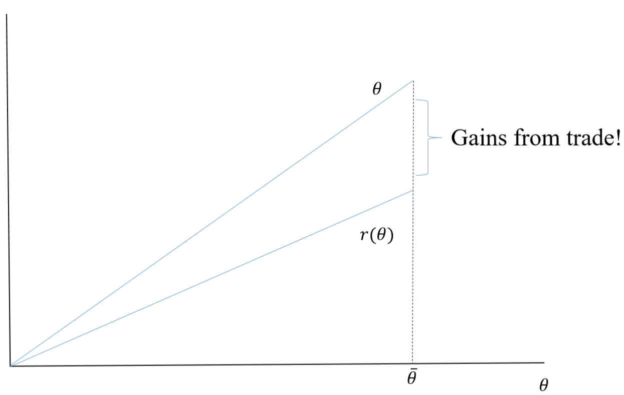

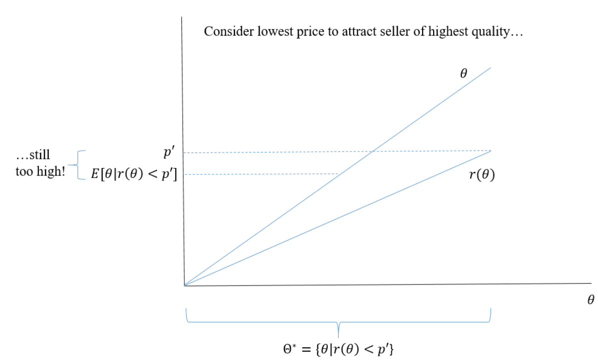

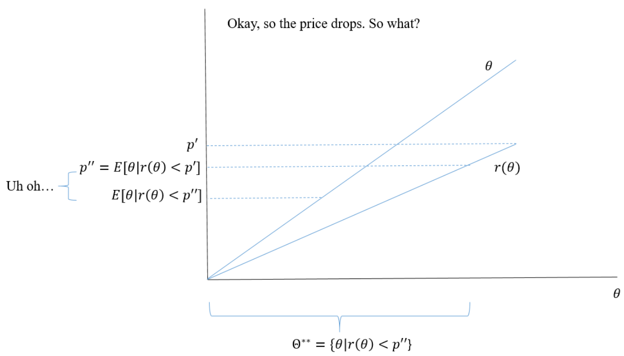

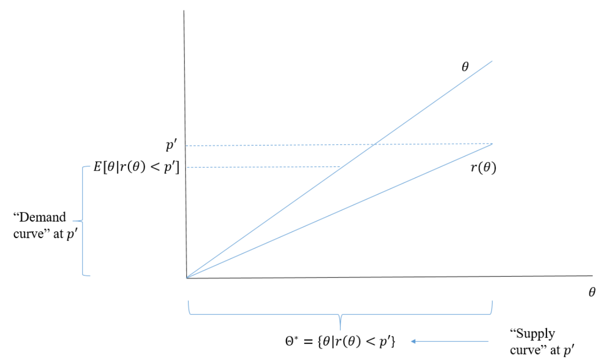

Start with the upper-left graph. The 45 degree line θ represents buyer valuations, while the lower line $r(\theta)$ represents sellers' valuations (i.e. "reservation" price). That sellers value the good less than buyers implies gains from trade. However, asymmetric information implies buyers know only the *average* quality of goods transacted.

The upper-right graph considers whether there could be an equilibrium with all goods sold. The lowest possible price of such an equilibrium is p^* (i.e. $r(\bar{\theta})$). However, it turns out that the average valuation of buyers is less than p^*, so the price cannot be that high.

The lower-left graph considers a different equilibrium where the price is lowered. This leads to some sellers dropping out of the market, which in turn lowers the average valuation of buyers. You can imagine many of these graphs showing the complete unraveling of the market.

The lower-right graph presents alternate intuition about each stage. For a given price, the number of sellers willing to transact is "supply", while the number of buyers willing to transact is "demand". The equilibrium condition then looks a lot like standard market analysis where we require "demand" = "supply".

1.2 Mathematical analysis

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Suppose $\theta \sim U[0,1]$ and $r(\theta) = \frac{2}{3}\theta$. Then for a proposed equilibrium p^*, buyer willingness to pay is:

$$E[\theta | r(\theta) \leq p^*]$$

Substituting the expression for $r(\theta)$:

$$E[\theta | \frac{2}{3}\theta \leq p^*]$$

Rearranging to make this in the familiar form for expectations:

$$E[\theta | \theta \leq \frac{3}{2}p^*]$$

It should be clear that uniformly distributed quality means that the average transacted good's quality will be half of the equilibrium price. Moreover, since the highest quality good is 1, this must be at most 1/2. Therefore, the above expression for willingness to pay is:

$$\min\{\frac{3}{4}p^*, \frac{1}{2}\}$$

.

The equilibrium therefore requires $p^* = \min\{\frac{3}{4}p^*, \frac{1}{2}\}$, which is uniquely satisfied by $p^* = 0$ (i.e. complete unraveling).

1.3 Additional adverse selection practice problem

In the market for used cars, there are high and low quality cars. At a price P, the quantity of high and low quality cars supplied are (in thousands of cars): $\max(2P - 8, 0)$ and $\max(P - 3, 0)$ respectively. To buyers, a low quality car is worth 4.5 and a high quality car is worth 7. Buyers are risk neutral, and if they are uncertain about the quality of the car, their valuation is the expected value given their beliefs about the quality. The number of buyers far exceeds the number of sellers, so the demand for cars is infinite if the price is less than the expected value of a car.

1. Suppose that buyers are perfectly informed about the quality of each car. What is the market price and quantity of high and low quality cars? You may find it helpful to graph the supply and demand curves. *Solution.* In the market for low quality cars, supply is $P - 3$ and consumers will purchases if $P \leq 4.5$. The equilibrium price is $P = 4.5$, with $Q = 1.5$.

   In the market for high quality cars, supply is $2P - 8$ and consumers are willing to pay 7. The equilibrium price is $P = 7$, with $Q = 6$.

   Prices won't be higher than these because then demand goes to 0. Prices will not be lowers because demand for cars is perfectly elastic.

2. Suppose now that buyers have no information about what kind of car they are purchasing. If the prevailing market price for cars is P, what fraction of the cars available for sale are high quality? At that price P, what are the buyers' expected valuation for the cars? *Solution.* At price P, there are $2P - 8$ high quality cars and $P - 3$ low quality cars, for a total of $3P - 11$ cars. To the buyer, any unknown car is high quality with probability $(2P - 8)/(3P - 11)$ (note the car is low probability with complementary probability $1 - (2P - 8)/(3P - 11) = (P - 3)/(3P - 11)$).

   If the buyer believes a car to be high quality with probability λ, his valuation is $7\lambda - 4.5(1 - \lambda)$. In this case, the quality is what we calculated above, and his expected value is $7 * (2P - 8)/(3P - 11) + 4.5 * (P - 3)/(3P - 11)$.

3. There is some price P such that the buyer's willingness to pay for a car is equal to price P. What is the quantity of high and low quality cars traded in this equilibrium? (Note: You will not be able to solve for the numbers by hand.) *Solution.*

   We are looking for some price P such that the market price leads buyers to be the same as the buyer's expected value (since demand is perfectly elastic). That is, we are looking to solve: $7 * (2P - 8)/(3P - 11) + 4.5 * (P - 3)/(3P - 11) = P$. Tedious algebra (or WolframAlpha) gives a quadratic with two solutions: $P = 5.92$ and $P = 3.90$. Note that the $P = 3.90$ is not a valid solution because that would correspond to a negative quantity of high quality cars sold, which is nonsensical.

4. Now suppose instead that the supply for high quality cars was $2P - 11$. If you were to solve part 3, you would find that there is no real solution to the quadratic function. What is the equilibrium in this market? Why does the solution change so much?
Solution. $P = 4.5$ is now an equilibrium where only low quality cars will be sold. Intuitively, there aren't enough high-quality cars to sustain high enough buyer WTP for high-quality cars to be sold.

2 Overview of Signaling Models

The above analysis related to market unraveling relied on one side of the market needing to make an inference about the other due to asymmetric information. In some sense, this was all passive based on willing to participate in the market or not; all market participants get pooled together. It makes sense that market participants might want to actively convey information to the other side to facilitate trade. This is the heart of signaling models.

Nonetheless, the analytical tools turn out to be the same: given a proposed equilibrium, you (1) deduce what one side of the market infers about the other and then (2) consider whether that's consistent with the proposed equilibrium.

2.1 Signaling problem

Let there be two ability types of programmers, High and Low, with abilities $\theta_H > \theta_L$ and prior probability of a high type $p \in (0, 1)$. A (verifiable) costly test can be taken; the cost of passing the test depends on ability: c_H for the high type and c_L for the low type, with $c_H < c_L$ (high types find the test less costly).

Firms observe only whether the candidate *passed* the test (signal $s \in \{\text{pass,fail}\}$) and pay a wage equal to the conditional expectation of ability given the observed signal and equilibrium beliefs. Denote by w_{pass} and w_{fail} the wages after observing pass and fail respectively.

1. Separating equilibrium (High pass, Low fail).
Actions (by assumption): High types pass, Low types do not.

Beliefs (Bayes):

$$\Pr(\text{High} \mid \text{pass}) = 1, \qquad \Pr(\text{High} \mid \text{fail}) = 0.$$

Wages therefore are

$$w_{\text{pass}} = \theta_H, \qquad w_{\text{fail}} = \theta_L.$$

Incentive constraints (ICs):

- High type must prefer passing:

$$(\text{pass payoff})\ \theta_H - c_H \ \geq \ (\text{not pass payoff})\ \theta_L \quad \Longrightarrow \quad c_H \leq \theta_H - \theta_L.$$

- Low type must prefer not passing:

$$(\text{not pass payoff})\ \theta_L \ \geq \ (\text{pass payoff})\ \theta_H - c_L \quad \Longrightarrow \quad c_L \geq \theta_H - \theta_L.$$

Thus a separating equilibrium with High passing and Low not passing exists if and only if

$$c_H \leq \Delta \leq c_L, \qquad \text{where } \Delta := \theta_H - \theta_L.$$

2. Pooling equilibrium (both do not pass).
Actions: both High and Low do not pass.

Beliefs after "fail": firms use prior, so

$$\Pr(\text{High} \mid \text{fail}) = p,$$

and the pooling wage (for those who do not pass) is the expected ability

$$w_{\text{pool}} = p\theta_H + (1-p)\theta_L = \theta_L + p\Delta.$$

(We assume off-path beliefs after "pass" do not support profitable deviations; we check deviations explicitly.)

Deviations:

- A High type would deviate (i.e., pass) if

$$\theta_H - c_H \ > \ w_{\text{pool}} \quad \Longleftrightarrow \quad c_H < \theta_H - w_{\text{pool}}.$$

- A Low type would deviate (pass) if

$$\theta_L - c_L \ > \ w_{\text{pool}} \quad \Longleftrightarrow \quad c_L < \theta_L - w_{\text{pool}}.$$

Because $\theta_L - w_{\text{pool}} = -p\Delta < 0$, the Low type never strictly gains by passing (since $c_L > 0$); the relevant constraint is the High type's:

$$c_H \geq \theta_H - w_{\text{pool}} = \Delta - p\Delta = (1-p)\Delta.$$

Therefore pooling on "fail" is an equilibrium if

$$c_H \geq (1-p)\Delta.$$

(If on-path beliefs and off-path wages are specified differently, other pooling equilibria may be supported by suitable off-path beliefs; the inequality above is the no-deviation condition under standard Bayesian updating and intuitive off-path beliefs.)

Summary of equilibrium regions.

- **Separating equilibrium** (High pass, Low fail) exists when

  $$c_H \leq \Delta \leq c_L.$$

  Intuition: high types can afford the test (relative to the benefit Δ), low types cannot.

- **Pooling equilibrium** (both do not pass) is sustained if

  $$c_H \geq (1-p)\Delta,$$

  i.e. when the high type's cost of passing is large enough that even the higher expected pooled wage does not justify paying the cost.

2.2 Additional signaling practice problem

Suppose I'm looking for someone to do data analysis as a UROP. The pool of undergrads consists evenly (i.e. 50%/50%) of two types: workaholics (w) and lazybones (l). The productivity of a workaholic is 10 regressions/day, while a lazybone produces only 4/day. Each regression gets me a $1 larger stipend (I wish!), and the department mandates that I pay my UROP's their expected productivity.

1. What's the wage in a pooling equilibrium?

   Solution. 7 (Wage in competitive pooling equilibrium is expected productivity)

2. David has offered to screen UROP's on my behalf. He administers a test that costs $1 to take, and he allows potential UROP's to take it multiple times. He's sneaky, so he keeps the profits and tells me only how many tests the potential UROP passed. Workaholics are studious, so they can always pass David's test. Lazybones slack off, so it always takes them two attempts to achieve one passed test.

   Question: Are two passed tests a credible signal that a worker is a workaholic? Why or why not?

   Solution. Two passed tests is not a credible signal. This is because in the separating equilibrium, the additional benefit from being known as a workaholic as opposed to a lazybones is $w^W - w^L = 10 - 4 = 6$

   Since to pass two tests, it costs the lazybones only $c^L(2) = 2 * 2 = 4$, so the lazybones take the tests (as do the workaholics).

3. What is the minimal number of passed tests that constitutes a credible signal of being a workaholic for the employer? *Solution.* The minimum credible number of passed tests is the one that just prevents the lazybones from taking the tests (and obtaining wage w^W instead of w^L). It is the e such that $c^L(e) \geq 6$, so $c^L(e) \geq 6 \Rightarrow 2e \geq 6 \Rightarrow e \geq 3$.

   So $e = 3$ is a sufficient number of tests to lead to a separating equilibrium. (Assuming that at $e = 3$, the lazybones will not take the tests as he is indifferent at that point, so we can assume only the workaholics are the ones taking the tests. Note that the highest number of tests the workaholic would take anyway is 6.)

4. Is signaling in the form of passing tests efficient from the point of view of society? *Solution.* In a separating equilibrium, the workaholics will be forced to take three tests. Because such tests do not improve the productivity of the workers, the tests are a waste of resources and Pareto inefficient. (However this is inefficient only compared to a world with perfect information!)