Exam Review 1

Salome Aguilar Llanes*

14.03 Fall 2025

Contents

1 Exam Structure

1. Short Questions
2. Consumer Theory
3. Empirical application

2 Exam Topics

- Minimum wage: perfect competition vs. monopsony
  - **Minimum wage: Perfect competition vs. monopsony**

    Perfect competition: Firms take the wage w as given. Profit maximization problem:

    $$\max_{L} \ \pi(L) = F(L) - wL$$

    First-order condition:

    $$F'(L) = w$$

    Equilibrium employment L^* satisfies MPL = wage.

    With a binding minimum wage $\bar{w} > w^*$, employment falls since labor demand decreases.

*Taking elements from lecture notes, recitations and the exam review of 2023

Monopsony: The firm faces an upward-sloping labor supply $w(L)$. Profit maximization problem:

$$\max_L \ \pi(L) = F(L) - w(L)L$$

First-order condition:

$$F'(L) = w(L) + L \cdot w'(L)$$

Without regulation, employment L^M is below the competitive level L^C. A binding minimum wage set at $\bar{w} \in (w^M, w^C)$ increases both wages and employment.

- Axioms of consumer preference

  1. **Completeness** – consumers have preference orderings over any two bundles of goods, every consumption bundle lies on some indifference curve
  2. **Transitivity** – consumers are *consistent*
  3. **Continuity** – indifference curves are smooth
  4. **Non-satiation** – consumers always prefer to have more of any given good (when holding the amount of other goods constant)
  5. **Diminishing MRS** – indifference curves are convex and utility function U(.) is concave

- Fundamental problem of causal inference: its not possible to see $Y_{1,i}$ and $Y_{0,i}$ at the same time. Workarounds:

  – Unit homogeneity
  – Temporal stability
  – Causal transience (reversibility)

  Don't work for people. We could compare people who take the treatment to people who don't but this is problematic

$$\Delta Y = E[Y_j(1) \mid T=1] - E[Y_j(0) \mid T=0]$$
$$= \Big(E[Y_j(1) \mid T=1] - E[Y_j(0) \mid T=1]\Big) + \Big(E[Y_j(0) \mid T=1] - E[Y_j(0) \mid T=0]\Big).$$

  The second term is the **selection bias**.

$$E[Y_j(0) \mid T=1] - E[Y_j(0) \mid T=0],$$

- **RCTs (setup, assumptions, and notation)** Randomized Controlled Trials assign treatment randomly to ensure independence between treatment and potential outcomes. Key assumption: randomization ensures unbiased estimation of treatment effects.

Under random assignment of treatment, we assume that we have

$$E[Y_j(0) \mid T = 1] - E[Y_j(0) \mid T = 0] = 0,$$

the selection bias term equals zero.

- **Difference-in-Differences (setup, assumptions, and notation)** Compares changes in outcomes over time between treatment and control groups. The key assumption is "parallel trends", in the absence of treatment, treated and control groups would have evolved similarly.

	Before	After	Change
Treatment	Y_{jb}	Y_{ja}	ΔY_j
Control	Y_{kb}	Y_{ka}	ΔY_k

  where b stands for before and a stands for after

- Formally, let's say that prior to treatment, we observe:

$$Y_{jb} = \alpha_j.$$
$$Y_{kb} = \alpha_k.$$

  We would hope that $\alpha_j \simeq \alpha_k$, but this does not strictly have to be the case.

- Now, imagine that after treatment, we observe

$$Y_{ja} = \alpha_j + \delta + T,$$

  where T is the causal effect and δ is any effect of time. For example, cholesterol levels may tend to rise over time as people age.

- So, if we take the first difference for Y_j, we get:

$$\Delta Y_j = Y_{ja} - Y_{jb} = \left(\alpha_j - \alpha_j\right) + \delta_j + T$$

  This does not recover T. But it does remove the "level effect" α_j.

- Similarly, let $\Delta Y_k = (\alpha_k - \alpha_k) + \delta_k$. Differencing removes the level effect for group j.

- If we are willing to postulate that the time effect operates identically on the treatment and control groups, $\delta_j = \delta_k = \delta$, then we have

$$\hat{T} = \Delta Y_j - \Delta Y_k = T + \delta - \delta.$$

- **RDDs (setup, assumptions, and notation)**

Regression Discontinuity exploits a cutoff in a running variable to identify causal effects. The key assumption is that units just above and below the cutoff are comparable, so any discontinuity in outcomes at the threshold identifies the treatment effect.

While arbitrary cutoffs are necessary for administration, they can be useful for economists. Define a variable X that is used to determine the cutoff above/below which a person (or unit) i is or is not assigned to treatment. For example, X could be the percentage of voters for candidate A or X could be the exact hour/minute/second of birth. We will refer to X as the *running variable*, and we'd like that variable to be continuous.

Imagine there are two underlying relationships between potential outcomes and treatment, represented by $E[Y_{i1}|X_i]$ and $E[Y_{i0}|X_i]$. Thus at each value of X_i, the causal effect of treatment is $E\left[T|X_i = x\right] = E[Y_{i1}|X_i = x] - E[Y_{i0}|X_i = x]$. Let's say that individuals to the right of a cutoff c (e.g., $X_i \geq 0.5$) are exposed to treatment, while those to the left $(X_i < 0.5)$ are denied treatment. We therefore observe $E\left[Y_{i1}|X_i\right]$ to the right of the cutoff and $E\left[Y_{i0}|X_i\right]$ to the left of the cutoff.

As we consider units i that are arbitrarily close (within ε) to the threshold, it may be reasonable to assume that:

$$\begin{aligned}
\lim_{\varepsilon \downarrow 0} E\left[Y_{i1}|X_i = c + \varepsilon\right] &= \lim_{\varepsilon \uparrow 0} E\left[Y_{i1}|X_i = c + \varepsilon\right], \\
\lim_{\varepsilon \downarrow 0} E\left[Y_{i0}|X_i = c + \varepsilon\right] &= \lim_{\varepsilon \uparrow 0} E\left[Y_{i0}|X_i = c + \varepsilon\right].
\end{aligned}$$

That is, for units that are *almost identical*, we may be willing to assume that had both been treated (or not treated), their outcomes would have been arbitrarily similar. If this assumption is plausible, we can form a Regression Discontinuity estimate of the causal effect of treatment on outcome Y using the contrast:

$$\hat{T} = \lim_{\varepsilon \downarrow 0} E\left[Y_i|X_i = c + \varepsilon\right] - \lim_{\varepsilon \uparrow 0} E\left[Y_i|X_i = c + \varepsilon\right],$$

which in the limit is equal to:

$$T = E\left[Y_{i1} - Y_{i0}|X_i = c\right].$$

The RD estimator estimates the causal effect of a treatment as the "jump" in an outcome variable, Y, as near-identical units on one side of a discontinuity, c, are allocated to treatment while those on the other side are allocated to non-treatment. Note that while RD estimation does estimate the treatment effect given that $x_i = c$, if the treatment effect is not the same for everyone, it will not give you the average treatment effect on the treated.

- Utility maximization (primal problem): Marshallian (uncompensated) demand, indirect utility function, Roy identity

$$\max_{x,y\geq 0} \quad u(x,y)$$
$$\text{s.t.} \qquad p_x x + p_y y \leq I$$

$$\text{Solution (Marshallian/uncompensated demands):} \qquad x = x(p_x, p_y, I), \quad y = y(p_x, p_y, I).$$

$$\text{Indirect utility:} \qquad V(p_x, p_y, I) := u\big(x(p_x, p_y, I),\, y(p_x, p_y, I)\big).$$

$$\text{Roy's Identity:} \qquad x(p_x, p_y, I) \;=\; -\frac{\partial V(p_x, p_y, I)/\partial p_x}{\partial V(p_x, p_y, I)/\partial I}, \qquad y(p_x, p_y, I) \;=\; -\frac{\partial V(p_x, p_y, I)/\partial p_y}{\partial V(p_x, p_y, I)/\partial I}.$$

- Cost minimization (dual problem): Hicksian (compensated) demand, expenditure function, Shephard's Lemma

Cost minimization (dual problem) & Slutsky equation

Consider prices (p_x, p_y) and a target utility level $\bar{u} > 0$.

$$\min_{x,y\geq 0} \quad E \;=\; p_x x + p_y y$$
$$\text{s.t.} \qquad u(x,y) \;\geq\; \bar{u}.$$

At the optimum the constraint binds, so we usually write $u(x,y) = \bar{u}$. Define the *expenditure function*

$$e(p_x, p_y, \bar{u}) \;=\; \min_{x,y\geq 0:\; u(x,y)\geq \bar{u}} \; p_x x + p_y y.$$

The (Hicksian) compensated demand functions are

$$h_x(p_x, p_y, \bar{u}), \qquad h_y(p_x, p_y, \bar{u}),$$

Shephard's Lemma. Under regularity,

$$\frac{\partial e(p_x, p_y, \bar{u})}{\partial p_x} = h_x(p_x, p_y, \bar{u}), \qquad \frac{\partial e(p_x, p_y, \bar{u})}{\partial p_y} = h_y(p_x, p_y, \bar{u}).$$

- **Relation between Marshallian and Hicksian demands.** Let $x_i(p_x, p_y, I)$ denote Marshallian (uncompensated) demand and $h_i(p_x, p_y, \bar{u})$ the Hicksian demand. For the income I that yields utility $\bar{u}$ (i.e. $I = e(p_x, p_y, \bar{u})$),

$$x_i(p_x, p_y, I) \;=\; h_i(p_x, p_y, \bar{u}) \quad \text{with } \bar{u} = V(p_x, p_y, I).$$

- Slutsky equation, substitution and income effects (normal, inferior and Giffen goods)

  Slutsky equation

$$\boxed{\frac{\partial x(p_x, p_y, I)}{\partial p_x} \;=\; \frac{\partial h_x(p_x, p_y, \bar{u})}{\partial p_x} \;-\; x(p_x, p_y, I)\,\frac{\partial x(p_x, p_y, I)}{\partial I}}$$

  where $\bar{u} = V(p_x, p_y, I)$ and $\dfrac{\partial x}{\partial I}$ is the income (wealth) derivative of Marshallian demand. Interpretation: total price effect = compensated (substitution) effect $-$ income effect (evaluated at the original income).

- The Carte Blanche principle When designing transfers or interventions, giving recipients *full discretion* (cash) is often valued more by recipients than restricting them to specific in-kind goods.

- Household Labor Supply with Home Production

  The household chooses consumption C, leisure L, market labor M, and home production H to maximize utility:

$$\max_{C, L, M, H} \quad U(C, L)$$

  subject to the constraints:

$$C = wM + C(H) \qquad \text{(consumption from wage income and home production)}$$

$$M + h + L \leq 24 \qquad \text{(time endowment)}$$

Earned Income Tax Credit (EITC)

- The EITC is a federal income subsidy for low-wage workers, specifically, a refundable tax credit.

- As of December 2023:

  – About 23 million eligible workers and families received the federal EITC.

  – Federal expenditures were $57 billion.

  – Average benefit per household was $2,541.

Federal EITC Benefits Schedule (2023)

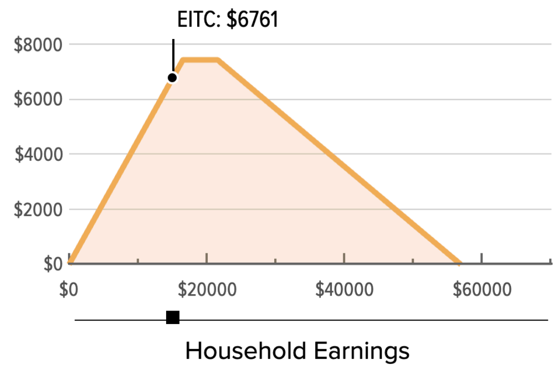
Single parent, three children

Value of Federal Earned Income Tax Credit, 2023

Filing Status:
Single/Head of Household

Number of Children:
Three or More

Household Earnings:
$ 15000



Note: Assumes all income is from earnings (as opposed to investments, for example).

Source: Internal Revenue Service

CENTER ON BUDGET AND POLICY PRIORITIES | CBPP.ORG