Recitation 6: The Edgeworth Box and Gains from Market Integration

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Introduction

In this recitation we will focus on the Edgeworth Box model, which economists use to model gains from market integration and interpersonal trade. So far, we have only treated demand as **endogenous**, while prices, income, and preferences are **exogenous**. However, in the Edgeworth box example, we endogenize prices to model where they come from. Recall this table from lecture:

	Consumer's Problem	**Partial Equilibrium**	**General Equilibrium**
Preferences	Exogenous	Exogenous	Exogenous
Budget set	Exogenous	Exogenous	**Endogenous**
Prices	Exogenous	**Endogenous**	**Endogenous**
Consumption	**Endogenous**	**Endogenous**	**Endogenous**

The consumer's problem is what you and I face every day: unfortunately, I can't control my preferences, my budget set (for the most part) or the prices of the goods around me. Thus, all I can do is react by changing my consumption (creating an endogenous demand function).

We relaxed this problem in the partial equilibrium, allowing prices to be endogenous as well for firms: by focusing on some market participants for a specific market, prices are now endogenous (e.g., wage being the price from a partial equilibrium in the labor market).

For market participants though, markets are interconnected, as they will *trade*. Then, we think about the General Equilibrium problem, and consider the simplest case of this– two goods, two agents– the **Edgeworth Box**.

The Edgeworth Box

Setup

We have two goods in this economy, coffee c and tea t. Now, we introduce our two agents to this economy: Emma (in red) and Nagisa (in blue). They each have some initial endowment of each, and since they are the only two agents in this economy, this is the full amount of coffee and tea that exists in this economy. They also have some utility function for each of these

- Emma's utility is $u_E(t_E, c_E) = \frac{1}{2}\ln t_E + \frac{1}{2}\ln c_E$
- Nagisa's utility is $u_N(t_N, c_N) = \frac{1}{4}\ln t_N + \frac{3}{4}\ln c_N$
- Emma starts with endowment $(E_E^t, E_E^c) = (3, 3)$
- Nagisa starts with endowment $(E_N^t, E_N^c) = (4, 1)$

Without any trade, Emmas's utility is

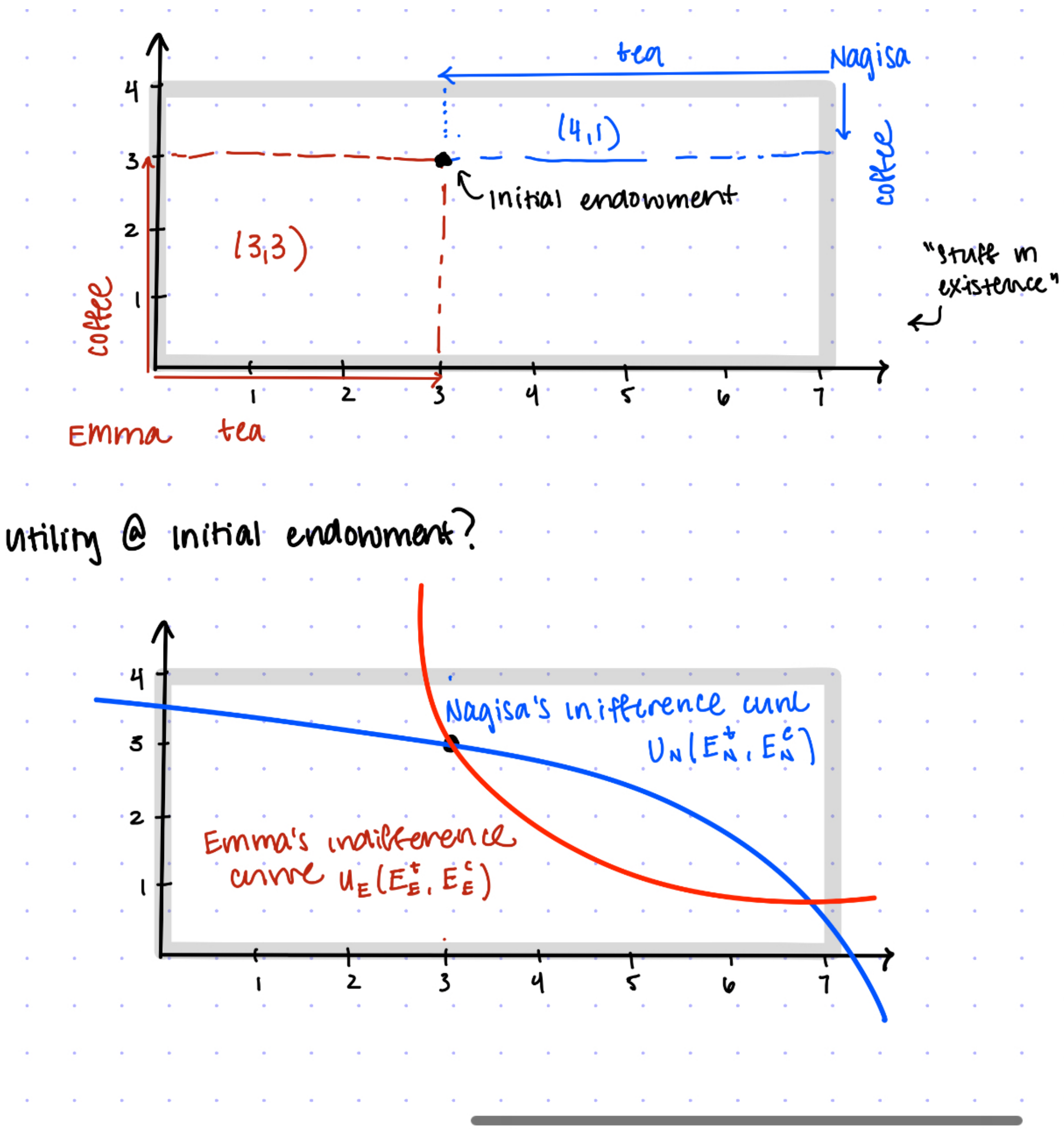
$$u_E(3,3) = \frac{1}{2}\ln 3 + \frac{1}{2}\ln 3 \approx 1.01$$

and Nagisa's utility is

$$u_N(4,1) = \frac{1}{4}\ln 4 + \frac{3}{4}\ln 1 \approx 0.35.$$

The figure below shows the setup of an Edgeworth Box in this context.

Figure 1: Endowments in the Edgeworth Box



We assume that Emma and Nagisa can trade according to the following assumptions:

1. No transaction costs
2. No market power (Agents take prices as given)
3. No externalities
4. Full information about the goods
5. Property rights (All goods are owned by somebody).
6. Consumer Theory Axioms

Finding Demands as a Function of Prices Using Lagrangians (Partial Equilibrium)

As discussed in class, any allocation on the contract curve could theoretically be acheived through **tattonnement**[1], meaning bargaining between Emma and Nagisa with potentially different price ratios at different times. However, when we solve the Edgeworth box using the Lagrangian method, we find the unique solution on the contract curve that can be reached given the initial endowment and a single, constant price ratio.

In other words, through tattonnement, they will always end up at a solution on the contract curve and within the original lens, but not necessarily the Lagrangian solution.

In order to solve using the Lagrangian method, we solve for Emma and Nagisa's demand as a function of prices p_t and p_c. This is very similar to the problems we solved in the consumer theory section - for the time being, we still treat prices and given. The only wrinkle is that the budget set is given by the consumer's endowment. Emma solves:

$$\max_{t_E, c_E} \left\{ \frac{1}{2} \ln t_E + \frac{1}{2} \ln c_E \right\} \quad \text{subject to} \quad p_t t_E + p_c c_E \leq 3p_t + 3p_c$$

and Nagisa solves:

$$\max_{t_N, c_N} \left\{ \frac{1}{4} \ln t_N + \frac{3}{4} \ln c_N \right\} \quad \text{subject to} \quad p_t t_N + p_c c_N \leq 4p_t + 1p_c.$$

Setting up both of our Lagrangians gives us:

$$\mathcal{L}_E(p_t, p_c, \lambda_E) = \frac{1}{2} \ln t_E + \frac{1}{2} \ln c_E - \lambda_E(p_t t_E + p_c c_E - 3p_t - 3p_c)$$
$$\mathcal{L}_N(p_t, p_c, \lambda_N) = \frac{1}{4} \ln t_N + \frac{3}{4} \ln c_N - \lambda_N(p_t t_N + p_c c_N - 4p_t - 1p_c).$$

[1]tattonnement: An iterative auction process by which an exchange equilibrium is imagined to be achieved. Walrasian auction is this iterative process but with an auctioneer

Taking derivatives, we get the following FOCs for Emma:

$$\frac{\partial \mathcal{L}_E}{\partial t} = \frac{1}{2t} - \lambda_E p_t = 0$$
$$\frac{\partial \mathcal{L}_E}{\partial c} = \frac{1}{2c} - \lambda_E p_c = 0$$
$$\frac{\partial \mathcal{L}_E}{\partial \lambda_E} = -p_t t_E - p_c c_E + 3p_t + 3p_c = 0$$

We can combine the first two FOCs to eliminate λ_E and get $t_E p_t = c_E p_c$, meaning Emma splits her budget evenly between tea and coffee. Combining that with the budget constraint, we get:

$$t_E^* = \frac{3p_t + 3p_c}{2p_t}$$
$$c_E^* = \frac{3p_t + 3p_c}{2p_c}$$

Similarly, for Nagisa we get $3t_N p_t = c_N p_c$, meaning she spends $\frac{3}{4}$ of her income on coffee and $\frac{1}{4}$ on tea. Combining with her budget constraint, we get the following Marshallian demands:

$$t_N^* = \frac{4p_t + p_c}{4p_t}$$
$$c_N^* = \frac{3(4p_t + p_c)}{4p_c}$$

Finding the Prices that "Clear" the Market (General Equilibrium)

We don't care about absolute prices, only relative prices. Things like taxes and inflation will affect both equally and Emma and Nagisa will end up purchasing the same amount. So, we want to solve for the price ratio p_c/p_t that clears the market. We use the fact that markets must clear: the total demand for tea must equal the total supply of tea. The same goes for coffee. For tea this means:

$$\begin{aligned}
\underbrace{t_E^* + t_N^*}_{demand} &= \underbrace{3+4}_{supply} \\
\frac{3p_t + 3p_c}{2p_t} + \frac{4p_t + p_c}{4p_t} &= 7 \\
\frac{3}{2} + \frac{3}{2}\left(\frac{p_c}{p_t}\right) + 1 + \frac{1}{4}\left(\frac{p_c}{p_t}\right) &= 7 \\
\frac{7}{4}\left(\frac{p_c}{p_t}\right) &= \frac{9}{2} \\
\frac{p_c}{p_t} &= \frac{18}{7}.
\end{aligned}$$

We could have done similar calculations for coffee , but we don't need to because we already have the one variable we're interested in: the price ratio. Plugging this price ratio into our demands, we get:

$$\begin{aligned}
t_E^* &= \frac{3p_t + 3p_c}{2p_t} = \frac{3}{2} + \frac{3}{2}\left(\frac{p_c}{p_t}\right) = \frac{3}{2} + \frac{3}{2}\left(\frac{18}{7}\right) = \frac{75}{14} \\
c_E^* &= \frac{3p_t + 3p_c}{2p_c} = \frac{3}{2}\left(\frac{p_t}{p_c}\right) + \frac{3}{2} = \frac{3}{2}\left(\frac{7}{18}\right) + \frac{3}{2} = \frac{25}{12} \\
t_N^* &= \frac{4p_t + p_c}{4p_t} = 1 + \frac{1}{4}\left(\frac{p_c}{p_t}\right) = 1 + \frac{1}{4}\left(\frac{18}{7}\right) = \frac{23}{14} \\
c_N^* &= \frac{3(4p_t + p_c)}{4p_c} = 3\left(\frac{p_t}{p_c}\right) + \frac{3}{4} = 3\left(\frac{7}{18}\right) + \frac{3}{4} = \frac{23}{12}
\end{aligned}$$

Plugging these into the utility functions, we find the equilibrium utilities is

$$u_E(\frac{75}{14}, \frac{25}{12}) = \frac{1}{2}\ln\frac{75}{14} + \frac{1}{2}\ln\frac{25}{12} \approx 1.21$$

and

$$u_N(\frac{23}{14}, \frac{23}{12}) = \frac{1}{4}\ln\frac{23}{14} + \frac{3}{4}\ln\frac{23}{12} \approx 0.61.$$

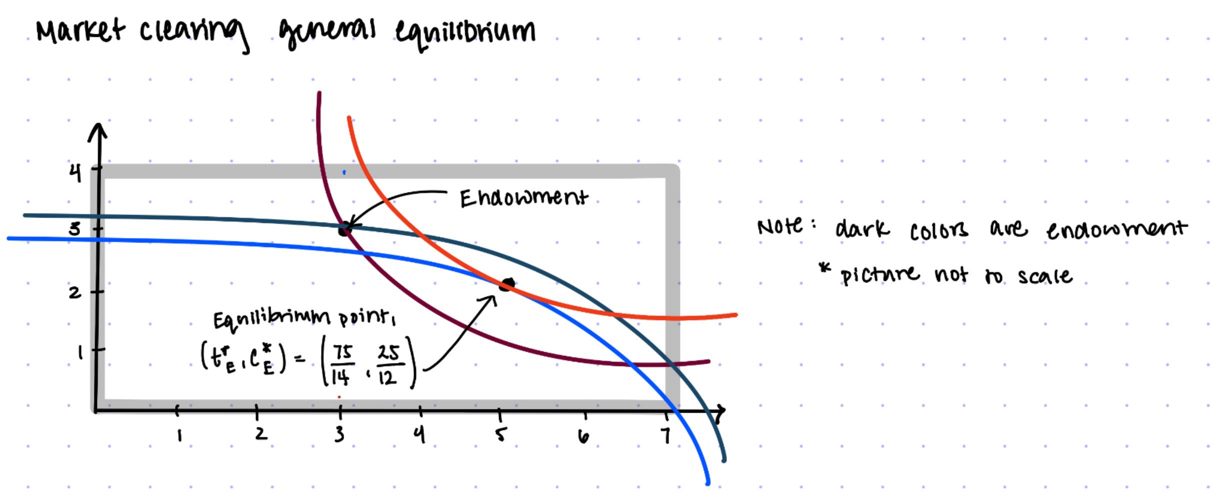
We see that both agents are made better off by trade! Emmas's utility increases from 1.01 to 1.21, and Nagisa's utility increases from 0.35 to 0.61.

- **Question**: Nagisa's utility increases by 0.26 utils, whereas Emmas's utility only increases by 0.2. Does this mean the trade is better for Nagisa?

- **Answer**: No! Remember we can never compare two people's utility functions. We just know that both Kate and Nagisa are better off than they were previously, but we still can't compare their welfare to each other.

The figure below shows the equilibrium in the Edgeworth Box.

Figure 2: Equilibrium in the Edgeworth Box



Finding The Contract Curve

Along the contract curve, we know that Emma's and Nagisa's indifference curves must be tangent (due to the first welfare theorem: equilibrium will be Pareto Optimal, thus the MRS is the same and they are tangent to each other), meaning their MRS must be the same.

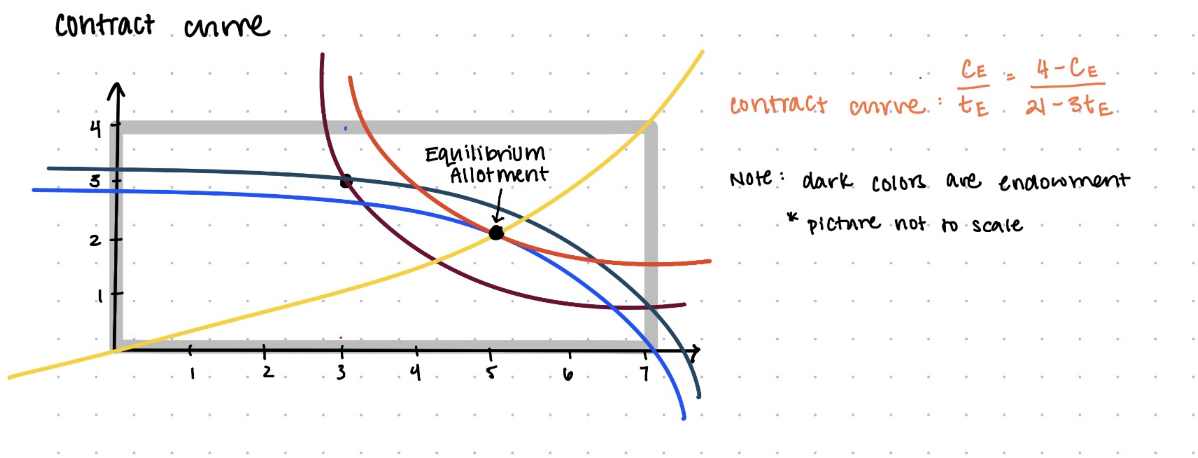
$$MRS_E = \frac{\partial u_E/\partial t}{\partial u_E/\partial c} = \frac{1/2t}{1/2c} = \frac{c}{t}$$
$$MRS_N = \frac{\partial u_N/\partial t}{\partial u_N/\partial c} = \frac{1/4t}{3/4c} = \frac{c}{3t}$$

Viewing the problem from Emmas's perspective, this would mean the contract curve is given by:

$$\frac{c_E}{t_E} = \frac{4 - c_E}{3(7 - t_E)}$$

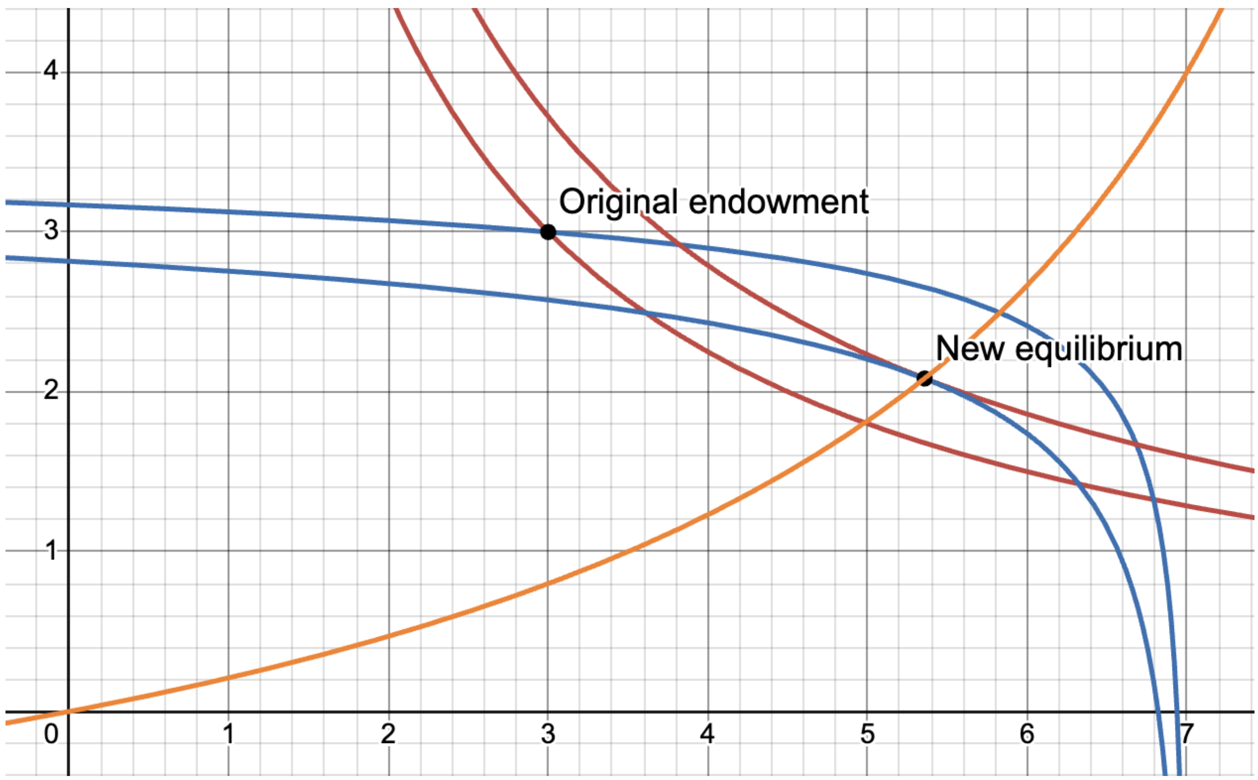
Adding this orange contract curve to the graph, we see that it goes throgh the bottom left corner, the top right corner, and our Lagrangian solution we found earlier!

Figure 3: Equilibrium with the Contract Curve



We can check our work by graphing with desmos:

Figure 4: Equilibrium with the Contract Curve



As we discussed in class, not all points on the contract curve are attainable given Emma's and Nagisa's original endowments. However, all are attainable given some reasonable original endowment.

- **Question:** If Emma and Nagisa started with different original endowments (but still a total of 7 teas and 4 coffees, would the Lagrangian solution arrive at the same price ratio as we found above?
- **Answer:** Nope! Remember that the price ratio is equal to the MRS of both Emma and Nagisa at the optimum. If, for example, one of them started out with way more tea than the other, their MRS would probably look quite different (due to our diminishing MRS requirement).

How to do policy?

We had a plickers question about "fairness" and "equity" in this setting, which it is probably not: most times, there will be some uneven distribution of goods and not every agent gets the same amount. Although these agents end up consuming similar amounts of coffee, Emma gets WAY more tea. A hypothetical government might see this, and, valuing equity, decide to step in.

lets say the governemnt wants to split things up equally at $(t_i, c_i) = (3.5, 2)$ Seems fair, right? But now Emma's utility is $u_E \approx 0.97$ and Nagisas is $u_N \approx 0.83$. That's terrible for Emma, even worse than her original endowment. She will surely start trading in order to be happier. And, seeing as Nagisa suddenly is drowning in coffee, she will accept. How do we change the equilibrium solution then?

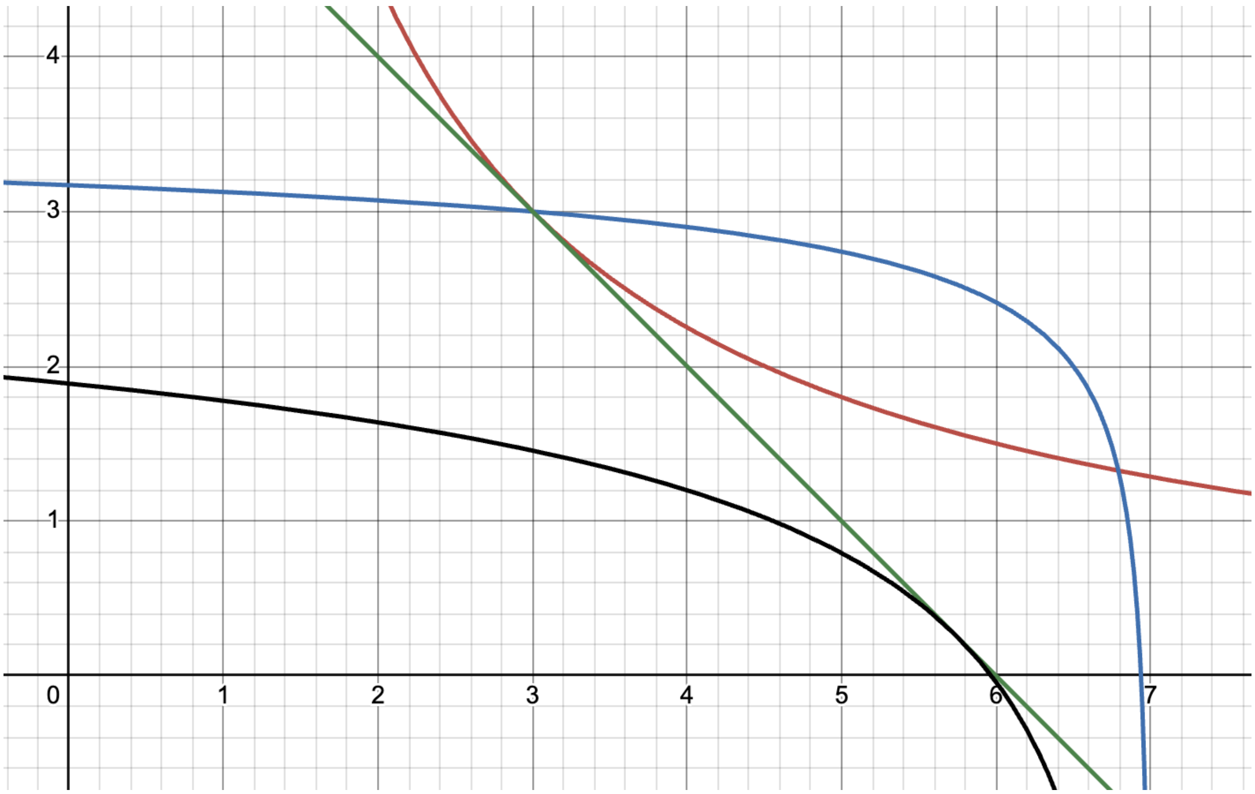
Price Ratio Manipulation

Since price ratio determines Pareto efficiency, one thought is to manipulate this price ratio by taxing one good and not the other. Seeing that Nagisa gets a lot less tea at equilibrium, the governement decides to start taxing tea and makes the price ratio $\frac{p_c}{p_t} = 1$. Now,

$$
\begin{aligned}
t_E^* &= \frac{3p_t + 3p_c}{2p_t} = \frac{3}{2} + \frac{3}{2}\left(\frac{p_c}{p_t}\right) = 3 \\
c_E^* &= \frac{3p_t + 3p_c}{2p_c} = \frac{3}{2}\left(\frac{p_t}{p_c}\right) + \frac{3}{2} = 3 \\
t_N^* &= \frac{4p_t + p_c}{4p_t} = 1 + \frac{1}{4}\left(\frac{p_c}{p_t}\right) = \frac{5}{4} \\
c_N^* &= \frac{3(4p_t + p_c)}{4p_c} = 3\left(\frac{p_t}{p_c}\right) + \frac{3}{4} = \frac{15}{4}
\end{aligned}
$$

At first it seems promising: Emma is weakly better off than the endowment conditions (she has equal utility) and Nagisa has massively benefited. There's one problem though...

Figure 5: Allocations under a new price ratio



Market clearing is not fulfilled. In fact the only way to have market clearing is along the contract curve. What then, is a government to do?

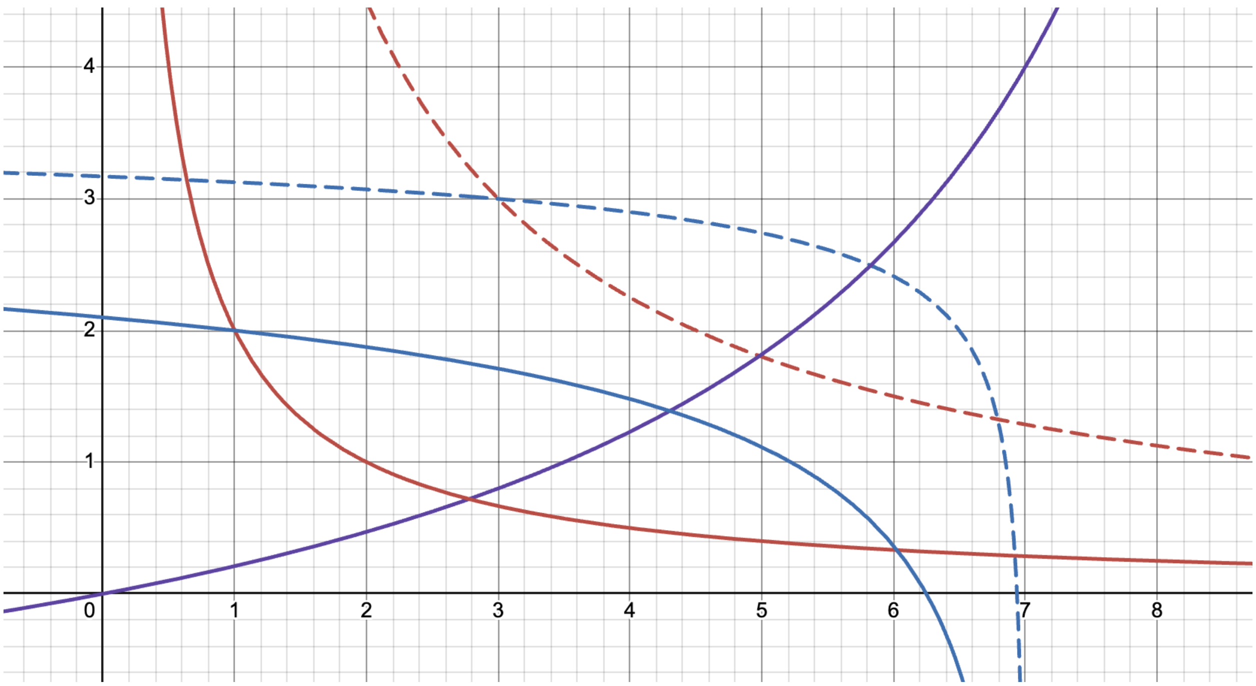
Also note that even if this price ratio was in the lens (and thus would be at least weakly better than the origional endowment), it is still not *optimal*. There is only one point where the price ratio = the marginal rate of substitution, and that is the efficient allocation. This is not to say that Nagisa and Emma wont trade at this point! However, they just have a *better* trade they can make. Since we are assuming full information, both of them know exactly how much the other will be willing to pay, so they will find this optimal price ratio.

A Lump Sum

The Second Welfare Theorem says that any Pareto efficient allocation– any allocation on the contract curve– is supportable as a market equilibrium. However, we had to get to that efficient allocation from our original endowment through some trading, meaning that in order for something to be better for both parties, the allocation has to be better than the endowment. Then how about we change the endowment (via a lump sum transfer?) Now the location of our lens has changed, and the optimal spot on the contract curve has too.

If Emma starts off with $(E_E^t, E_E^c) = (1, 2)$ and Nagisa with $(E_N^t, E_N^c) = (6, 2)$ the **Pareto Efficient** region has changed.

Figure 6: The new endowment



Now solving for consumption:
Setting up both of our Lagrangians gives us:

$$\mathcal{L}_E(p_t, p_c, \lambda_E) = \frac{1}{2}\ln t_E + \frac{1}{2}\ln c_E - \lambda_E(p_t t_E + p_c c_E - 2p_t - 1p_c)$$
$$\mathcal{L}_N(p_t, p_c, \lambda_N) = \frac{1}{4}\ln t_N + \frac{3}{4}\ln c_N - \lambda_N(p_t t_N + p_c c_N - 2p_t - 6p_c).$$

Solving these yields

$$\boxed{r = \frac{10}{3}, \quad t_E^* = \frac{23}{6},\ c_E^* = \frac{23}{20}, \quad t_N^* = \frac{19}{6},\ c_N^* = \frac{57}{20}}$$

With a price ratio of $\frac{p_c}{p_t} = 10/3$, which indeed intercepts the contract curve at the correct point.