Recitation 7: International Trade and Instrumental Variables

Notes adapted from a previous year's recitation by Kate Ellison

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Trade

A Baking Example

A professional baker and a novice cook each attempt to bake bread and make food.
The professional baker produces according to the following production functions:

$$B_b = 4\sqrt{H_B}$$
$$F_b = 2\sqrt{H_F}$$

Where H_B represents hours spent baking bread and H_F represents hours spent cooking food. The professional baker 20 hours in a week to devote to working. To derive the professional baker's PPF, we first invert the production functions:

$$H_B = \frac{B_b^2}{16}$$
$$H_F = \frac{F_b^2}{4}$$

And then plug these into the budget constraint:

$$\frac{B_b^2}{16} + \frac{F_b^2}{4} \leq 20$$

The PPF describes all bundles that could possibly be produced. But how do we know which bundle actually will be produced? For this, we would need some idea of preferences between output goods. That sounds kind of like... a utility function! Let's assume that the baker has a utility for bread and food given by

$$U(B_b, F_b) = ln(B_b) + ln(F_b)$$

We solve this using a Lagrangian like we did when we talked about utility maximization.

$$\max_{B_b,F_b} ln(B_b) + ln(F_b) \quad \text{subject to} \frac{B_b^2}{16} + \frac{F_b^2}{4} \leq 20$$

We can calculate the MRS between bread and food as:
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$$\frac{1}{B_b} = \lambda \frac{B_b}{8}$$

$$\frac{1}{F_b} = \lambda \frac{F_b}{2}$$

Solving, we get:

$$-\frac{F_b}{B_b} = \frac{B_b}{4F_b}$$

$$4F_b^2 = B_b^2$$

Plugging this back into the PPF, we get:

$$\frac{4F_b^2}{16} + \frac{F_b^2}{4} \leq 20$$

.

$$\frac{2F_b^2}{4} \leq 20$$

.

$$F_b^2 \leq 40 = 4 * 10$$

. Solving, we get $F_b^* = 2\sqrt{10}$ and $B_b^* = 4\sqrt{10}$.

If we were interested in resource allocation, we could look back at our original production functions and see that the professional baker spends 10 hours baking and 10 cooking food.

Finally, in many trade problems, you will be asked about the price ratio between two different goods. As when we studied utility theory, at the optimum, price ratio should be equal to the marginal rate of substitution. Using our MRS from above, we get:

$$\frac{MU_B}{MU_F} = \frac{p_B}{p_F}$$
$$\frac{p_B}{p_F} = \frac{F_b}{B_b}$$
$$= \frac{2\sqrt{10}}{4\sqrt{10}}$$
$$= \frac{1}{2}$$

When we think of the example, prices don't quite make as much sense as when we consider material goods. However, the price ratio still has a meaningful interpretation. A price ratio of $\frac{p_B}{p_F} = \frac{1}{2}$ means that the professional baker would trade 2 completed breads for 1 dish.

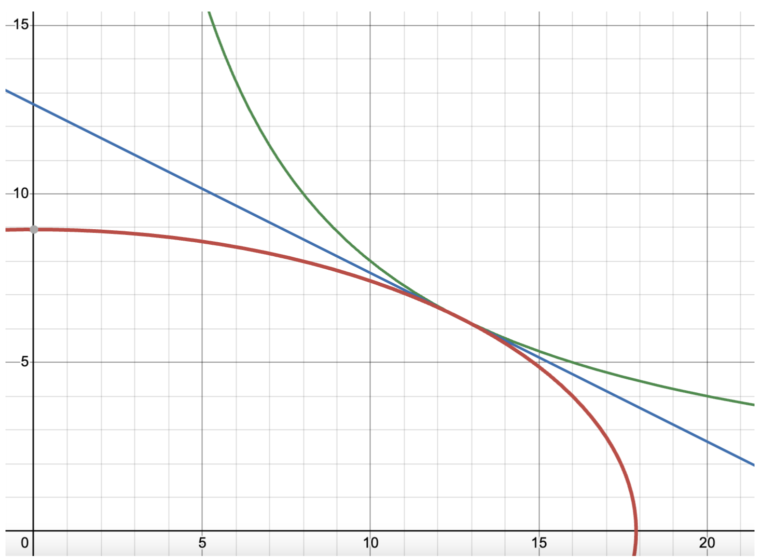

Figure 1: Equilibrium Under Autarky

Introducing Trade

Now, let's consider the fact that the world not only has a professional baker, but also has a novice cook who bakes bread and cooks food. Their production function is given by:

$$B_c = \sqrt{H_B}$$
$$F_c = \sqrt{H_F}$$

Note that the novice cook is strictly worse at baking bread and cooking food than the professional baker. This means that the professional baker has an **absolute advantage** at baking and cooking. However, we should also note that the professional baker is twice as efficient at baking, while the novice cook is exactly as efficient at baking bread and cooking food.

The price ratio of the novice cook's is $\frac{p_B}{p_F} = 1$.

Now let's explore trade! For the purposes of this question, assume there are infinite number of novice cooks and they have infinite hours to spend cooking and baking (so any "trade" they do with the professional baker doesn't change the price ratio).

In order to find what the professional baker would do under trade, we solve for the point on the professional baker's PPF that intersects with the new price ratio of 1.

We remember that the derivative of the PPF is given by

$$-\frac{dF}{dB} = \frac{B_b}{4F_b}$$

Setting this equal to 1, we get

$$\frac{B_b}{4F_b} = 1$$
$$B_b = 4F_b$$

Plugging this back into the professional baker's PPF, we get:

$$\frac{(4F_b)^2}{16} + \frac{F_b^2}{4} = 20$$

Solving, we get $F_B^P = 4$, $B_B^p = 16$. This is how much the professional baker will now produce. But how much will they consume? How will they trade with the novice cooks? In order to figure this out, we want to find the highest indifference curve that is is tangent to the new, including-trade PPF.

The first step is to equate the MRS with the price ratio. As found above, we know the MRS is given by:

$$\frac{MU_B}{MU_F} = \frac{F_b}{B_b}$$

Equating this with the price ratio of 1, we get $F_b = B_b$. The "new" PPF under trade is given by the line of slope 1 that goes through the production point of $F_b^p = 4$, $B_b^p = 16$.

The equation for such a line would be given by $E = 20 - P$. Solving this system of two equations, we get $F_b^c = 10$, $B_b^C = 10$.

This concludes the math-heavy part of an international trade problem, but let's take a step back and consider the bigger implications of trade problems. This problem presents a nice opportunity to think about comparative advantage. The novice cooks are simply worse at production of both bread and food. However, that doesn't mean it doesn't make sense for the professional baker to trade with them.

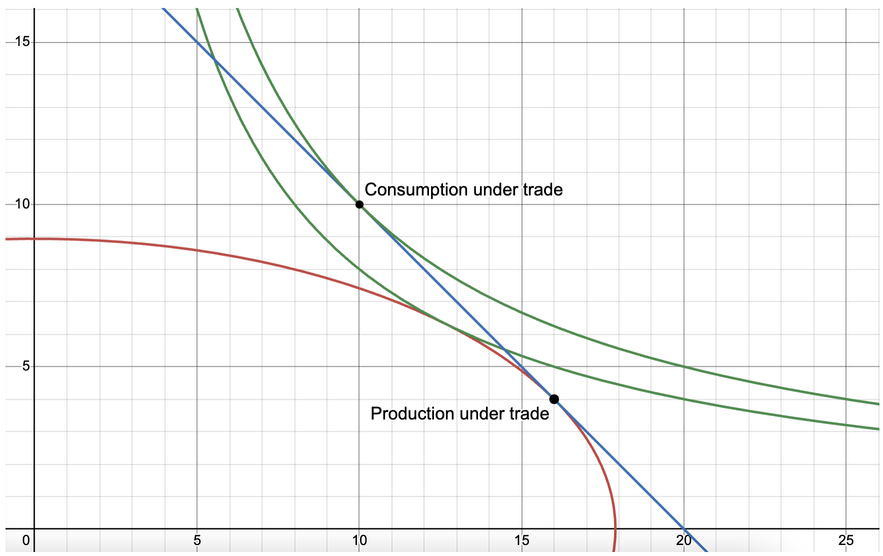

Figure 2: Equilibrium Under Trade

Instrumental Variables

General IV Info

In class, we learned that instrumental variables (IV) provide a powerful tool to estimate causal effects when the variable of interest is endogenous. Often, the challenge is that the treatment X is correlated with unobserved characteristics that also affect the outcome Y, making simple regression estimates biased.

An IV design helps isolate variation in X that is plausibly exogenous. We use an instrument Z that affects X but has no direct effect on Y other than through X:

$$Z \to X \to Y$$

The instrument must satisfy two key assumptions:

- **Relevance:** Z must affect the endogenous variable X.
- **Exclusion restriction:** Z must affect Y only through X, not through any other channel.

Motivating Example: Online Courses and Student Success

Let's look at an example from the paper *Virtual Classrooms: How Online College Courses Affect Student Success* by Eric Bettinger, Lindsay Fox, Susanna Loeb, and Eric S. Taylor.

The authors study how taking a college course online, rather than in-person, affects students' academic performance and persistence in college. As online learning becomes increasingly common, understanding its impact on student outcomes is crucial for both universities and policymakers.

The main outcomes studied are students' grades in the course taken, grades in future courses, and the likelihood of remaining enrolled at the university. The causal question is straightforward:

Does taking a course online (X) affect student success (Y)?

However, comparing online and in-person students directly would likely be misleading. Students self-select into online courses; those who choose them might differ systematically in motivation, time constraints, or prior achievement. This creates an endogeneity problem: unobserved characteristics could drive both course modality and performance.

Instrument: Distance × Online Availability

To address this challenge, the authors use an instrumental variable based on the interaction between a student's **distance from campus** and whether the **university offers the course online.**

The idea is intuitive: Students who live farther from campus are more likely to take an online version if it is available.

The instrument, therefore, captures exogenous variation in students' likelihood of taking an online class due to logistical constraints rather than individual preferences or ability.

Assessing the IV Assumptions

First Stage (Relevance). The instrument is relevant because when universities offer a course online, students who live farther away become significantly more likely to take the online version. This variation in modality choice is exactly what we need to predict online enrollment. Empirically, the authors show that distance interacted with online availability strongly predicts whether a student takes the course online.

Exclusion Restriction. The exclusion restriction requires that the instrument affects student outcomes only through its effect on taking an online course. This seems plausible: distance from campus should not directly affect a student's grade in a course if both in-person and online options are equally accessible, except through its influence on whether the student chooses the online format. Similarly, the decision by the university to offer a course online should not directly affect a given student's performanceâexcept by changing the modality that student experiences.

Interpretation

Under these assumptions, the IV estimates capture the *Local Average Treatment Effect (LATE)*: the causal effect of taking a course online for the group of students whose enrollment decision (online vs. in-person) is influenced by the instrument, that is, those whose modality choice depends on distance and course availability.

The results show that, for these students, taking a course online reduces success: grades are lower in the online course, grades in future courses decline, and students are less likely to remain enrolled. In other words, for students with access to both formats, online courses have negative effects on academic performance and persistence.