Final Exam

Problem 1. Labor supply of secondary earner (45 minutes)

Consider a couple, Amy and Bob, with the following utility function

$$U = log(c_A + c_B) + log(l_A) + log(l_B)$$

where c_A, l_A denote Amy's consumption and leisure and c_B, l_B denote Bob's consumption and leisure. Each family member is endowed with $T = 60$ hours of time. If Amy chooses to work, she faces hourly wage w_A. If Bob chooses to work, he faces hourly wage $w_B < w_A$. Suppose the price for the consumption goods is 1. Suppose the family files tax jointly and faces flat income tax rate τ.

1. Derive the family labor supply function h^* as function of w_A, w_B and τ. What is Bob's elasticity of labor supply with respect to wage w_B, evaluated at the optimal level of labor supply? Denote your answer as η_h^B.

Answer: the budget constraint is

$$c_A + c_B + w_A(1 - \tau)l_A + w_B(1 - \tau)l_B = 60w_A(1 - \tau) + 60w_B(1 - \tau)$$

Let $C = c_A + c_B$. The optimization problem is

$$max \ log(C) + log(l_A) + log(l_B)$$

$$s.t. \ c_A + c_B + w_A(1 - \tau)l_A + w_B(1 - \tau)l_B = 60w_A(1 - \tau) + 60w_B(1 - \tau)$$

FOCs yield $l_A = 20\frac{w_A + w_B}{w_A}$, $l_B = 20\frac{w_A + w_B}{w_B}$ and $C = c_A + c_B = 20(1 - \tau)(w_A + w_B)$.

We have $h_B^* = 60 - l_B = 60 - 20\frac{w_A + w_B}{w_B} = 40 - 20\frac{w_A}{w_B}$ and $\eta_B = \frac{\partial h^*}{\partial w_B} \cdot \frac{w_B}{h^*} = \frac{w_A}{2w_B - w_A}$. Similarly, $h_A^* = 60 - l_A = 60 - 20\frac{w_A + w_B}{w_A}$. So $h^* = 120 - 20(\frac{w_A + w_B}{w_A} + \frac{w_A + w_B}{w_B})$.

2. The birth of a new baby complicates the couple's labor supply decisions. Continuing with the setup in part 1, suppose a total of 20 hours of childcare is required. The childcare can be provided by the family (either Amy or Bob), or be outsourced to a childcare center at price c per hour. First write down the family's optimization problem and budget constraint for three possible childcare arrangements.

a. Amy works and Bob stays at home to take care of the baby.

b. Bob works and Amy stays at home to take care of the baby.

c. Both Amy and Bob work; Hire a childcare provider to take care of the baby.

Without doing any math, which arrangement do you think is the worst (in terms of maximizing family utility)? Which arrangement do you think is the best? How does your answer depend on τ and c and why?

Answer: [Note: you can assume either (1) the care-taker allocates the remaining 40 hours of his/her time between labor and leisure, or (2) the care-taker spends all 40 hours in leisure. The solution here is based on (1).] For (a), the budget constraint is

$$c_A + c_B = w_A(1 - \tau)(60 - l_A) + w_B(1 - \tau)(60 - l_B - 20)$$

For (b), the budget constraint is

$$c_A + c_B = w_A(1 - \tau)(60 - l_A - 20) + w_B(1 - \tau)(60 - l_B)$$

For (c), the budget constraint is

$$c_A + c_B + 20c = w_A(1 - \tau)(60 - l_A) + w_B(1 - \tau)(60 - l_B)$$

(b) is clearly worse than (a): Amy is more productive than Bob at work, since $w_A > w_B$. The comparison between (a,b) and (c) depends on the relative magnitudes of c v.s $(1 - \tau)w_A$ and $(1 - \tau)w_B$:

- If $c < (1 - \tau)w_B < (1 - \tau)w_A$, (c) is the best and (b) is the worst;
- If $(1 - \tau)w_B < c < (1 - \tau)w_A$, (a) is the best and (b) is the worst;
- If $(1 - \tau)w_B < (1 - \tau)w_A < c$, (a) is the best and (c) is the worst.

3. Suppose they have decided to have Bob take care of the child for t_c hours. Consider the following two proposals to assist the family in covering the cost of child care:

   a. A tax change, allowing parents with total family income below a certain level $\bar{Y}$ to deduct all child care expenses from their taxes;

   b. A voucher of amount $10c$, eligible for parents with total family income below a certain level $\bar{Y}$.

   Write down the family's new budget constraint under each proposal. Without doing any math, explain intuitively how each proposal will affect the labor supply decision. Which proposal would you recommend and why? Does your answer depend on the cutoff $\bar{Y}$?

   Answer: In the absence of any program, the budget constraint is

   $$c_A + c_B + w_A(1 - \tau)l_A + w_B(1 - \tau)l_B + w_B(1 - \tau)t_c = 60w_A(1 - \tau) + 60w_B(1 - \tau)$$

   With tax deduction in (a), the budget constraint is

   $$c_A + c_B + w_A(1 - \tau)l_A + w_B(1 - \tau)l_B + w_B(1 - 2\tau)t_c = 60w_A(1 - \tau) + 60w_B(1 - \tau)$$

   With voucher in (b), the budget constraint is

   $$c_A + c_B + w_A(1 - \tau)l_A + w_B(1 - \tau)l_B + w_B(1 - \tau)t_c = 60w_A(1 - \tau) + 60w_B(1 - \tau) + 10c$$

   The proposal in (a) is equivalent to a price reduction of t_c, which has both income and substitution effects (the solid black line). The proposal in (b) is a pure income effect (the dotted line). The answer will depend on the cutoff (see graph below).

4. Your economist friend published a paper rejecting the family-utility model above. She argues that each parent values his/her own consumption and leisure, as well as the child's well-being (captured by the **total** time

spent on childcare). Specifically, the utility functions of Amy and Bob are given by, respectively

$$U_A = log(c_A) + log(l_A) + log(T_A + T_B)$$

$$U_B = log(c_B) + log(l_B) + log(T_A + T_B)$$

where T_A represents the amount of time Amy devotes to childcare and T_B represents the amount of time Bob devotes to childcare.

a. Solve for the socially optimal level of $T \equiv T_A + T_B$ (i.e., the level of T that maximizes the sum of U_A and U_B) .

    Answer: [You may assume either (1) the family has a pooled budget constraint or (2) Amy and Bob has two separate budget constraints. Solution here is based on (1).]The social optimum is solution to

$$max\ log(c_A) + log(l_A) + log(c_B) + log(l_B) + 2log(T_A + T_B)$$

$$s.t.\ c_A + c_B + (1-\tau)w_A l_A + (1-\tau)w_B l_B + (1-\tau)w_A T_A + (1-\tau)w_B T_B = (1-\tau)60(w_A + w_B)$$

Setting MRS to price ratios, we have

$$l_A^* = 10\frac{w_A + w_B}{w_A}$$

$$T_A^* = 20\frac{w_A + w_B}{w_A}$$

$$l_B^* = 10\frac{w_A + w_B}{w_B}$$

$$T_B^* = 20\frac{w_A + w_B}{w_B}$$

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b. Solve for the equilibrium level of T if Amy and Bob make decisions independently. Compare your answer to part 4(a) and explain the difference (if any).

    Answer: Amy's problem is

$$max\ log(c_A) + log(l_A) + log(T_A + T_B)$$

$$s.t.\ c_A + (1-\tau)w_A l_A + (1-\tau)w_A T_A = (1-\tau)60 w_A$$

    which yields $l_A = 20$, $T_A = 20$ and $c_A = 20(1-\tau)w_A$.

Solving Bob's problem in the same way we have $l_B = 20$, $T_B = 20$ and $c_B = 20(1-\tau)w_B$.

Note that $T_A^* > T_A$ and $T_B^* > T_B$, which is the classic free-riding problem.

Problem 2. Taxation and Externality (39 minutes)

In the market for football ticket, the demand is $Q = 50 - 3P$ where P is price (in thousand-dollars) and Q is number of ticket sold (in thousands). Suppose the supply for ticket is $Q = 2P$ until the stadium is at its full capacity of 18,000 seats.

1. Due to the increasing number of noise complaints from residents near the stadium, the government decides watching football games imposes a negative consumption externality and taxes consumers $\frac{5}{3}$ dollars for every ticket purchased.

    a. Graph the supply and demand curves and label the pre-tax market price. Why does the supply curve have the specific shape?

        Answer: Pre-tax equilibrium is given by the intersection of $Q = 50 - 3P$ and the supply curve, which is $P^* = 10$ and $Q^* = 18$ (the intersection is on the vertical portion of the supply curve). The after-tax equilibrium is given by the intersection of $Q = 50 - 3(P + \frac{5}{3})$ and the supply curve, which gives producer price $P = \frac{32}{3}$ and $Q = 18$. The supply curve is vertical after $Q = 18$ likely due to stadium capacity.

    b. Using the pre-tax market price, calculate the burden of the tax borne by each party. (You may use either the exact formula or the elasticity-based approximation formula to calculate the tax incidence).

        Answer: Since both the pre- and post- tax equilibrium intersects the perfectly-inelastic part of the supply curve, the supplier bears 100% of the tax.

    c. Suppose that the stadium doubles in capacity. How will your answer to part b change? Explain the intuition.

        Answer: Now pre-tax equilibrium is given by the intersection of $Q = 50 - 3P$ and the supply curve, which is $P^* = 10$ and $Q^* = 20$. The after-tax equilibrium is given by the intersection of $Q = 50 - 3(P + \frac{5}{3})$ and the supply curve, which gives producer price $P = 9$ and $Q = 18$.

$$\eta_S = \frac{dQ}{dP} \cdot \frac{P}{Q} = 2\frac{P}{2P} = 1$$

$$\eta_D = -3\frac{P}{50 - 3P} = -\frac{3}{2}$$

Consumer burden is

$$\frac{\eta_S}{\eta_S - \eta_D}\tau = \frac{2}{3}$$

Producer burden is

$$\frac{\eta_D}{\eta_S - \eta_D}\tau = 1$$

For the remaining parts of this question, assume the capacity of the stadium has been expanded, as in part 1c.

2. Since football is such a wonderful game, the government wants it to be affordable to everyone. Hence, prior to implementing the tax, the government had capped the price at $P = 8$. Now, once the tax is implemented, how does the tax incidence change from part 1? Explain your answer intuitively.

    Answer: Pre-tax equilibrium with price-ceiling is $P = 8$ and $Q = 16$. Post-tax equilibrium with price-ceiling depends on whether the ceiling is placed on producer price or consumer price. If the ceiling applies to

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consumer price, producers bear the full tax burden. If the ceiling applies to producer price, consumers bear the full burden. .

3. Football teams often threaten to move to other cities, and cities respond by spending public money on building new stadiums. Evaluate the implications of the cities' responses on local and national welfare.

   Answer: Correct answer highlights that local effect is uncertain (depends on how much they pay and spillvoers to lcoal economy) but the national effect is clearly negative.

4. A new study found a strong causal relationship between brain disease and deaths of professional football players. Public health experts argue that football games impose an enormous negative externality on the players and hence should be banned altogether. Do you agree with the experts' assessment? How does your answer depend on the uncertainty of the potential health damage?

   Answer: This is not an externality if football players are fully aware of and fully compensated for the potential health damage. However if the players are not fully aware of the potential damage, this could be a large internality which calls for intervention. If the cost of potential health damage is certain, price tools usually work better than quantity regulation. Suppose the cost of potential health damage is uncertain (so the marginal benefit of reducing football games is uncertain). (a) If the marginal cost is steep, quantity regulation is better (size of blue triangles becomes small as MC becomes vertical). (b) If the marginal cost is flat, price scheme is better (size of orange triangles becomes small as MC becomes horizontal).

Problem 3. Student loans (41 minutes)

Consider a college graduate who earn income w_H if she finds a high-paying job and earn income w_L if she only finds a low-paying job. Suppose the probability of finding a high-paying job is p. Suppose she financed her college tuition bill S (in dollars) by taking out a 100% coverage tuition loan. Suppose the loan is an income-specific flat amount: she owes T_H in debt if she gets a high-paying job and T_L in debt if she gets a low-paying job.

Let $u(c)$ denotes the individual's utility from consuming c in a given state. Suppose that the government's student loan program is required to be ex-ante budget-balanced - that is, the expected amount of money collected in payments from the student must equal the overall amount of loan made to the student.

1. Write down the government's budget constraint.

   Answer: $pT_H + (1 - p)T_L = S$

2. Suppose that a student's consumption in a given state is equal to the wage she earns, minus the debt she pays, plus the amount of college tuition. That is, we assume that tuition payments deliver consumption value - perhaps because a higher tuition implies a more enjoyable college experience. Write down a student's expected utility.

   Answer: $p \cdot u(w_H + S - T_H) + (1 - p) \cdot u(w_L + S - T_L)$

3. Taking college tuition bill S as given, solve for the optimal level of T_H and T_L. [Hint: the math will be easier if you work in terms of the value of tuition net of debt repayments, e.g. define $R_H = T_H - S$ and $R_L = T_L - S$.] Which is larger, T_H or T_L? Why?

   Answer: Define $R_H = T_H - S$ and $R_L = T_L - S$ and rewrite the objective and constraint as

$$\max_{R_H, R_L} p \cdot u(w_H - R_H) + (1 - p) \cdot u(w_L - R_L)$$

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$$p \cdot R_H + (1-p)R_L = 0$$

Plugging in government budget constraint, rewrite expected utility as

$$p \cdot u(w_H + \frac{1-p}{p}R_L) + (1-p) \cdot u(w_L - R_L)$$

Optimal R_L will satisfy $w_H + \frac{1-p}{p}R_L = w_L - R_L$. Equivalently,

$$T_L = S - p(w_H - w_L)$$

$$T_H = S + (1-p)(w_H - w_L)$$

Note that $T_H > T_L$, which makes sense because the tuition debt structure serves as insurance.

4. Now, suppose the probability of finding a high-paying job is not exogenously given. Specifically, suppose p is a function of T_L.

   a. What do you think is the sign of $\frac{dp}{dT_L}$?

      Answer: $\frac{dp}{dT_L} > 0$. A higher loan repayment T_L will cause the student to search harder for a high-paying job.

   b. Without doing any math, explain how the optimal level of T_H and T_L will change relative to your answer to part 3.

      Answer: Intuitively, there will be moral hazard and the optimal insurance will be less than full. The precise expressions for T_H and T_L are derived as following. The budget constraint is

      $$R_H = -\frac{(1-p)}{p}R_L$$

   Differentiate to obtain

   $$\frac{dR_H}{dR_L} = -\frac{1-p(R_L)}{p(R_L)} - \frac{\frac{dp(R_L)}{dR_L}R_L}{(p(R_L))^2}$$
   $$= -\frac{1-p(R_L)}{p(R_L)} - \frac{\eta}{p(R_L)}$$

   where $\eta = \frac{dp(R_L)}{dR_L}\frac{R_L}{p(R_L)}$ is the elasticity of p with respect to R_L. Write student's objective function as

   $$V(R_H, R_L) \equiv \max_p p \cdot u(w_H - R_H) + (1-p) \cdot u(w_L - R_L)$$

   By FOC,

   $$\frac{\partial V/\partial R_L}{\partial V/\partial R_R} = -\frac{dR_H}{dR_L}$$

   or

   $$\frac{(1-p)u'(w_L - R_L)}{pu'(w_H - R_H)} = \frac{1-p}{p}(1 + \frac{\eta}{1-p})$$

   $$\frac{u'(w_L - R_L) - u'(w_H - R_H)}{u'(w_H - R_H)} = \frac{\eta}{1-p}$$

   Since $\eta > 0$ per part a, now only partial insurance is optimal (moral hazard effect).

5. Due to a recent recession, the government's student loan program now has massive default rate. Lawmakers are debating policy proposals to decrease tuition support. The three proposals are as follows.

- No change in tuition support;

- Reducing tuition support to 80% of S;

- Reducing tuition support to 50% of S.

In addition, there are three voting blocs.

- Group A prefers less tuition support to more.

- Group B prefers the middle-ground option, followed by the status quo option.

- Group C prefers no change to the status quo, but should any change be made, they would prefer a large reduction in tuition support relative to a small reduction.

Which group's preference is single-peaked? Assuming each group accounts for 1/3 of the voters, will majority voting generate consistent outcomes? Explain why or why not.

Answer: Group A and Group B are single-peaked. Group C's preferences are multi-peaked, since the middle option is least preferred. Majority voting will not generate consistent outcomes: there will be cycling.

Proble 4.1 T/F/U (40 minutes)

1. A research study found that corporations that finance their investments with a larger ratio of debt to equity tend to pay higher rates of interest to lenders. This finding is likely due to the fact that debt-heavy firms tend to hire managerial teams who are less risk-averse.

Answer: False. As the fraction of firm financing that is debt rises, the potential for conflict of interest between equity v.s debt holders grows: the debt holders bear an increasing share of projects that go bankrupt, while equity holders have a smaller and smaller risk from taking a gamble. So banks will charge higher interest rates on loans to firms as their share of debt financing rises. These higher interest rates offset the tax advantage of debt.

2. The elasticity of demand for yachts is much higher than that for cars, so the government should tax cars much more highly than yachts.

Answer: False. The statement ignores equity concerns: luxuries are mostly consumed by the rich while necessities are most heavily consumed by the poor.

3. The tension between equity and efficiency associated with the Hall-Rabushka flat-tax proposal can be ameliori-ated by increasing the exemption level and the rate of the flat tax.

Answer: Uncertain. For reasonable exemption level and tax rate, a flat tax is still much less progressive for high-income earners. If the tax rate is set too high, we risk efficiency loss.

4. Compare two countries that are identical except for one thing. In the first, no one wants to leave money to their kids, and any money left behind upon death is just by accident. In the second country, everyone wants to leave money to their kids, so money left behind is intentional. The estate tax should be higher in the second country than the first.

Answer: False. Accidental bequests means that people did not intend to leave bequests (they wanted to spend their wealth on themselves or they accumulated wealth because they enjoyed it). In that case, taxing inheritances does not distort work, savings and wealth accumulation behavior of those leaving bequests. It affects inheritors who receive less and make them work more (through income effects), which is also desirable (unless the society puts high weights on the utility of the inheritors).

5. Raising the contribution limit for tax-subsidized retirement savings account such as IRA will likely encourage retirement saving.

Answer: False. For high savers, this will be a pure income effect, which will lead to more first-period consumption. For low savers, the income and substitution effects go in opposite directions and the net effect is ambiguous.

6. The empirical evidence on unemployment spell durations suggests that workers who leave unemployment earlier (that is, find or take a job sooner) have no higher post unemployment wages than do workers who leave unemployment later. Hence the time window of coverage of UI should be shortened.

Answer: Uncertain. If the lack of improved job-math quality is due to moral hazard, then the statement is correct. If people who are unemployed for longer period suffer from skill depreciation which makes it harder for them to find good jobs, then the statement is false. If it is harder for people who are unemployed for long period of time to self insure, the statement can be false (consumption smoothing benefit outweighs moral hazard cost).

7. Economists have found that individuals' consumption declines in the first year of retirement. The lack of full consumption smoothing proves that individuals are only partially insured against leaving the labor force.

Answer: False. Full insurance means equating marginal utilities across time, not equating consumption across time. Individuals may have a higher marginal utility of consumption when working relative to in retirement, for example, if labor and consumption are complements in the utility function.

8. The current application process for disability insurance is extremely lengthy. Rules that will make it easier for workers to apply for and receive disability benefits will be welfare-improving.

Answer: False. This can be interpreted as the "ordeal mechanism": by making the application process lengthy and painful, we prevent the non-deserving from applying to DI.

Problem 4.2 Short Answers (12 minutes)

Suppose cigarette smoking imposes a negative externality of $ 1 per pack.

a. Provide a reason why cigarette should be taxed at more than $1/pack.

Answer: If in addition to externality, internality (e.g self-control problem) is also present.

b. Provide a reason why cigarette should be taxed at less than $1/pack.

Answer: If there's redistributional concerns (cigarette smokers tend to be from the lower part of the income distribution), or if there's general equilibrium effect (taxing cigarette causes substitution toward other more socially costly behaviors e.g drug/alchohol abuse).

c. Provide a reason why cigarette should be taxed at exactly $1/pack.

Answer: If the negative externality of $1 per pack is the only source of distortion.