14.41/14.410 Midterm Exam

Solutions and Grading Comments

Fall 2017

Problem 1 (25 minutes)

True/False/Uncertain. Provide brief justifications.

1. Caffeine is a highly addictive substance found in coffee, tea, and some soda. Therefore, its consumption should be regulated or taxed.

Answer: True if you think there are internalities (c.f., smokers' self-control problem) or negative externalities (e.g., caffeine over consumption leads to disruptive behaviors), but otherwise false.

2. Voters rarely get to choose the exact level of spending on a public good. Instead, they are provided with two options—a proposed spending level posed by the government and a default (or "reversion") level that would be enacted if the proposal were rejected by voters. This behavior is consistent with a size-maximizing government.

Answer: True/uncertain. Government can achieve size-maximization by manipulating (1) the default level to something much below the desired level of spending and (2) the proposed spending level to something a little greater than the desired level of spending. To avoid this extremely undesirable default, voters would have to choose the proposed spending level, which is larger than optimal.

3. The opportunity cost of a government purchase varies depending on whether the market for the purchased good is perfectly competitive or monopolistic. In the absence of cost concessions, a monopolist supplier will charge the government at a markup, making the project more costly.

Answer: False. It's opportunity cost that should be factored into cost-benefit analysis. Monopolist rent/profit/mark-up is just a transfer.

4. Stratmann (1995) documented a condition of "logrolling" in Congress, in which members of Congress trade votes on one bill for votes on another. Logrolling should be banned since it is abuse of power by Congress members seeking to maximize their own benefits.

Answer: False/uncertain. Logrolling could be efficient if it allows expression of preference intensity. For example, a minority group with intensive preferences for a bill might not get their bill passed under a single-majority voting system. But by logrolling, they can trade favors with a majority group to win their votes. If the minority group's preferences are sufficiently strong, such exchange of favors can be efficient.

5. To evaluate the effectiveness of vouchers in improving educational attainment, we can offer a given number of vouchers to any student in a particular town on a first-come-first-serve basis and compare the educational performance of the student receiving vouchers with those who do not receive vouchers.

Answer: False. Those who are most eager to get the vouchers are likely to be those who have strong preferences for education (which will over-estimate the effect of the vouchers) or those who come from disadvantaged households (which will under-estimate the effect of vouchers).

Problem 2 (25 minutes)

[Note: partial credit will be given for correct intuition, even if you cannot figure out the math.]

Two siblings, Amy and Bob both enjoy drinking coffee (C) and watching Netflix (N). Their utility functions are, respectively

$$U_A = ln(C_A) + 2 \cdot ln(N_A)$$

$$U_B = ln(C_B) + ln(N_B)$$

Let $p_C = 1$ be the unit price of coffee (\$/cup) and $p_N = 2$ the unit price of Netflix (\$/day). Let the total budget on coffee and Netflix be $I = \$20$.

Living in the same household, Amy and Bob share their Netflix account. Specifically, if one person pays for 1 day of Netflix, then the other person can also enjoy 1 day's Netflix for free. Hence, the actual amount of Netflix that one consumes is the sum of two individual quantities: $N = N_A + N_B$.

1. Suppose their parents care equally about both of them. What will be the level of coffee and Netflix consumption their parents choose for Amy and Bob?

Answer: The parents' optimization problem is

$$max\ ln(C_A) + ln(C_B) + 3 \cdot ln(N)$$

s.t $p_C(C_A + C_B) + p_N N = 20$. We know at optimum, $C_A = C_B = \frac{C}{2}$ (we discussed this in recitation 2). So the problem can be written as

$$max\ 2ln(C/2) + 3 \cdot ln(N)$$

or equivalently

$$max\ 2ln(C) + 3 \cdot ln(N)$$

s.t $p_C C + p_N N = 20$. Setting MRS to price ratio, we have

$$\frac{3C}{2N} = \frac{p_N}{p_C} = 2$$

which yields $N = 6$ and $C_A = C_B = \frac{C}{2} = 4$.

2. Suppose their parents give \$10 each to Amy and Bob and tell them to choose their own coffee and Netflix consumption individually. What will be the total level of coffee and Netflix consumption? Note Amy and Bob still live in the same household and share the Netflix account.

Answer: Amy's optimization problem is

$$max\ ln(C_A) + 2 \cdot ln(N_A + N_B)$$

s.t $p_C C_A + p_N N_A = 10$. Setting MRS to price ratio, we have

$$\frac{N_A + N_B}{2C_A} = \frac{p_N}{p_C}$$

Or $C_A = \frac{p_N(N_A+N_B)}{2p_C}$. Plugging C_A into the budget constraint, we have

$$p_C \frac{p_N(N_A+N_B)}{2p_C} + p_N N_A = 10$$

Or

$$N_A = \frac{20 - p_N N_B}{3p_N} = \frac{10 - N_B}{3}$$

We obtain a symmetric expression for Bob as

$$N_B = \frac{10 - p_N N_A}{2p_N} = \frac{5 - N_A}{2}$$

Combine the last two equations to get

$$N = N_A + N_B = 4 < 6$$

which is less than the answer to part 1. This is the classic free-riding problem.

3. Suppose their parents want to put either Amy or Bob in charge of coffee and Netflix spending. The person in charge will maximize the joint utility function with a personal bias. Specifically, if Amy is in charge of family finance, she will maximize $U = U_A + \frac{1}{2}U_B$ subject to the family budget constraint. On the other hand, if Bob is in charge of family finance, he will maximize $U = U_B + \frac{1}{2}U_A$ subject to the family budget constraint. Suppose that you are the parents of Amy and Bob. Who would you prefer to be in charge of the decision and why? How does your preferred outcome compare to parts 1 and 2 and why?

Answer: The maximization problem for Amy is

$$max\ ln(C_A) + 2 \cdot ln(N_A + N_B) + \frac{1}{2}ln(C_B) + \frac{1}{2}ln(N_A + N_B)$$

s.t. the pooled budget constraint. Let $C = C_A + C_B$. We know Amy will choose $C_A = \frac{2}{3}C$ and $C_B = \frac{1}{3}C$. Her objective can be rewritten as

$$max\ ln(\frac{2}{3}C) + 2 \cdot ln(N) + \frac{1}{2}ln(\frac{1}{3}C) + \frac{1}{2}ln(N)$$

Or equivalently,

$$max\ \frac{3}{2}ln(C) + \frac{5}{2}ln(N)$$

s.t $C + 2N = 20$. Setting MRS to the price ratio, we have

$$\frac{3N}{5C} = \frac{1}{2}$$

Or $N = \frac{25}{4}$, $C = \frac{15}{2}$, and the value of the objective function is $U = ln(10) + ln(\frac{5}{2}) + \frac{5}{2}ln(\frac{25}{4}) = 7.8$. The maximization problem for Bob is

$$max\ \frac{1}{2}ln(C_A) + ln(N_A + N_B) + ln(C_B) + ln(N_A + N_B)$$

s.t. the pooled budget constraint. Let $C = C_A + C_B$. We know Bob will choose $C_A = \frac{1}{3}C$ and $C_B = \frac{2}{3}C$. His objective can be rewritten as

$$max\ \frac{1}{2}ln(\frac{1}{3}C) + 2 \cdot ln(N) + ln(\frac{2}{3}C)$$

Or equivalently,

$$max\ \frac{3}{2}ln(C) + 2 \cdot ln(N)$$

s.t $C + 2N = 20$. Setting MRS to the price ratio, we have

$$\frac{3N}{4C} = \frac{1}{2}$$

Or $N = \frac{40}{7}$, $C = \frac{60}{7}$, and the value of the objective function is $U = \frac{1}{2}ln(\frac{20}{7}) + 2ln(\frac{40}{7}) + ln(\frac{40}{7}) = 5.8$. So Amy should take care of family finance. Intuitively, this is so since Amy's objective is more aligned with the social planner's: she cares more about N than Bob does, so putting her in charge will partly solve the free-riding problem.

Problem 3 (30 minutes)

[Note: partial credit will be given for correct intuition, even if you cannot figure out the math.]

Anna and Ben each owns an oil company on the gulf coast. If Anna chooses to invest x_A units of resources in oil exploration and produce d_A barrels of oil, her production cost is

$$TC_A(d_A, x_A) = \frac{1}{2}d_A^2 + (x_A - 2)^2$$

Oil exploration yields an additional benefit if the identification of the location of oil allows for others to drill for oil more effectively. In particular, suppose Ben's total cost depends on the number of barrels of oil he drills d_B as well as Anna's investment in oil exploration:

$$TC_B(d_B, x_A) = \frac{1}{2}{d_B}^2 + 2d_B - x_A d_B$$

Assume oil sells in a perfectly competitive market for $p = \$2$ per barrel.

Accidents on oil rigs, which occur with probability $\delta = 0.6$, cause spills which damage the inhabitants of the gulf states. In the event of an accident, the combined value of damage to residential properties, long-term health, etc. is estimated to be \$2 per barrel. Suppose the damage can be costlessly recouped through the legal system. Assume all parties involved are risk neutral.

1. Identify all sources of externalities and classify them as positive, negative or both. Determine whether the Coase theorem applies.

Answer: Anna's investment in oil exploration x_A imposes a positive production externality on Ben, since Ben's marginal cost of production is decreasing in x_A. Coase theorem may not apply since it might be difficult to assign property rights to an oil field after it is identified. Note that if the property owners damaged by oil spills can costlessly recoup costs through the legal system, as stated in the question, then the oil companies will internalize the legal penalty into its private costs, so there will be no externality. Students may argue that long-term health costs are uncertain and cannot be fully incorporated by the oil companies, in which case there will be a negative production externality.

2. What is the equilibrium amount of resources that Anna will invest in oil exploration x_A? Given Anna's choice of x_A, find the equilibrium level of Ben's production d_B.

Answer: Anna solves

$$\max_{d_A, x_A} 2d_A - \frac{1}{2}d_A^2 - (x_A - 2)^2$$

which yields $x_A = 2$ and $d_A = 2$. Given $x_A = 2$, Ben solves

$$\max_{d_B} 2d_B - \frac{1}{2}{d_B}^2 - 2d_B + 2d_B$$

will choose $d_B = 2$.

3. What is the socially optimal level of d_A, d_B and x_A? Compare your answers from part 2 and 3.

Answer: We solve

$$\max_{d_A, d_B, x_A} 2d_A + 2d_B - \frac{1}{2}d_A^2 - (x_A - 2)^2 - \frac{1}{2}{d_B}^2 - 2d_B + x_A d_B$$

which yields

$$d_A = 2$$

$$-2(x_A - 2) + d_B = 0$$

$$2 - d_B - 2 + x_A = 0$$

Combine the last two lines to get

$$x_A = 4$$

$$d_B = 4$$

4. Now suppose that the inhabitants of the gulf states **cannot** recoup any of the cost through the legal system. How does your answer to part 3 change? Explain the intuition.

Answer: The social marginal benefit of oil drilling is $SMB = 0.4 \times \$2 + 0.6 \times \$0 = \$0.8$. The socially efficient level of production is found by maximizing (by replacing price $p = PMB = 2$ from part 3 with SMB)

$$0.8d_A + 0.8d_B - \frac{1}{2}d_A^2 - (x_A - 2)^2 - \frac{1}{2}d_B{}^2 - 2d_B + x_A d_B$$

which yields

$$d_A^* = 0.8$$

$$-2(x_A^* - 2) + d_B^* = 0$$

$$0.8 - d_B^* - 2 + x_A^* = 0$$

Combine the last two lines to get

$$x_A^* = 2.8$$

$$d_B^* = 1.6$$

5. Let's continue with the situation in part 4, where the cost cannot be recouped. Intuitively, what combination of two government policy instruments could move us towards the social optimum? Mathematically, can you calculate the optimal level of government intervention to be implemented?

Answer: Comparing the equilibrium from part 1 and 4, we see

$$x_A^* > x_A$$

So oil exploration should be subsidized for \$0.8 per unit. On the other hand, $d_A^* + d_B^* < d_A + d_B$, so oil production should be taxed, and the amount of tax per unit is determined by the difference between private marginal benefit (\$2) and the social marginal benefit (\$0.8), which equals to \$1.2.