Midterm

14.41 Public Finance and Public Policy

85 Minutes Allowed

Instructions

- Use a different blue book for responses to each section (Question 1, Question 2, Question 3)
- Please write neatly; illegible work will be given no points.
- No calculators are allowed.

Question One [16 points]

For each question state whether the claim is **true, false or uncertain** and explain why. You must give reasons or no marks will be awarded.

1. Climatologists are sure that the marginal social benefit of each tonne of carbon dioxide abated is \$50, while the private marginal benefit is 0. Economists are unsure about the marginal cost functions of the firms that would actually do the abating.

   Claim: In this situation a quantity restriction is the best way for the government to reduce deadweight loss.

   False. Given the cost uncertainty it is not clear what the socially optimal quantity would be to mandate, so a quantity restriction would likely cause some deadweight loss. A price intervention, such as a tax set to \$ 50, would ensure the deadweight loss is zero no matter what the cost functions turn out to be. This is driven by the assumption of a constant net marginal social benefit of \$50.

   1 for false, 2 for realizing that generically when the marginal damage is close to constant a tax is a good idea, 1 for specifically realizing here that that deadweight loss would be zero with the right tax no matter the cost uncertainty. (Other answers could get some marks , but they are too varied to enumerate completely here).

2. A park is being planned for Somerville. The cost will be \$1,000. It will be financed by asking what each of the 100 local residents are willing to pay for this park, and charging them each the number they report. Assume these are the only benefits or costs that matter. Everyone reports \$ 11, and so the estimated net benefit is \$1100.

   Claim: given these reports, the park should be built.

   True. For a public good, we cannot trust that everyone will report their true WTP, but we can be sure they will under-report it. So if WTP$>$MC even with this under-reported \$11 WTP, then with the true WTP the park is an even better deal, and should proceed.

   2 for saying something correct about how elicitations are not to be trusted naively, 2 for translating that consistently and specifically into a conclusion about whether it should go ahead.

3. Claim: In order to redistribute from the advantaged to the disadvantaged, education should be provided by local governments.

   False. There is an efficiency motive to provide education locally, but redistributive motives would push in the direction of national provision so that money can be reallocated between local government areas. Since much of the inequality is between local government areas and not within them, redistribution is best done at a higher level of government than local.

   Referencing the tibout model got a few marks, noting that this scheme can just lead to rich towns and poor towns. Also making some reference to inequality between vs within towns was important.

4. The government is considering investing in a public works project that would produce \$10 in social benefits. The project can be completed using one unit of a special material made by a monopolist. It costs the monopolist \$5 but is sold to the government for \$15.

Claim: The government should not undertake the project.

False. The economic cost of the materials is $5, so the net economic benefit is $5. The $10 markup is just a transfer from the government to the monopolist, so it should not be considered in the cost/benefit analysis.

Noting the true cost is only $5

Research and Development [30 points]

Alice and Bob have pursued their lifelong goals of becoming successful inventors, and today they each manage a large firm. Firms A (Alice) and B (Bob) sell widgets to consumers in competitive markets, and initially assume that each firm caters to different consumers: Consumers in the market for widget A would never buy widget B, and consumers in the market for widget B would never buy widget A.

Consumers are willing to pay more for higher quality widgets, and Alice and Bob can improve their widgets (and earn higher profits) by employing scientists who do research and development (R&D). Scientists can be employed at the perfectly competitive wage w, and let l_A and l_B denote the number of scientists employed by Alice and Bob, respectively. The scientists at each company often publish papers about their research and talk to other scientists at conferences, exchanging ideas and knowledge. As a result, if Alice and Bob employ l_A and l_B scientists, the total profits earned by firms A and B from the sale of their widgets are

$$\pi_A(l_A, l_B) = 4(l_A l_B)^{1/4} - w l_A \quad \text{and} \quad \pi_B(l_A, l_B) = 4(l_A l_B)^{1/4} - w l_B.$$

Suppose that Alice and Bob separately choose the number of scientists to employ to maximize their profits, taking the number employed at the other company as given.

1. (4 pts) Assuming a positive number of scientists are employed, how many scientists do Alice and Bob employ in equilibrium?

   The interior first order condition for l_A is

   $$l_B^{1/4} l_A^{-3/4} - w = 0.$$

   Solving yields

   $$l_A = l_B^{1/3} w^{-4/3}.$$

   The solution for l_B is symmetric, so the (symmetric) equilibrium employment level satisfies

   $$l_e = l_e^{1/3} (w)^{-4/3} \iff l_e = w^{-2}.$$

   1 point - set up profit maximization problem, 1 point - first order condition, 2 points - solution

2. (7 pts) What employment levels are socially optimal? Explain why these are not achieved in equilibrium.

   The interior first order condition for l_A is

   $$2 l_B^{1/4} l_A^{-3/4} - w = 0.$$

   Solving yields

   $$l_A = l_B^{1/3} \left(\frac{w}{2}\right)^{-4/3}.$$

The solution for l_B is symmetric, so the (symmetric) optimal employment level satisfies

$$l_* = l_*^{1/3}\left(\frac{w}{2}\right)^{-4/3} \iff l_* = \left(\frac{w}{2}\right)^{-2}.$$

Employment is lower in equilibrium because each firm does not internalize the positive externality that its R&D has on the other firm: The scientists at firm B are more productive (hence more valuable) if there are more scientists employed at firm A. This positive production externality is not internalized in equilibrium.

1 point - set up welfare maximization problem, 1 point - first order condition, 2 points - solution, 3 points - explanation

3. (4 pts) Describe a possible Coasean solution to this externality. How does the feasibility of this solution depend on the number of firms undertaking R&D?

   Alice and Bob could enter into a "joint venture" and jointly optimize over the number of scientists each of them employs. Equivalently, they could bargain over the number of scientists to employ and split the surplus arbitrarily between themselves. These solutions are infeasible for large numbers of firms because transactions costs become unreasonably high or because of free-riding.

   2 points - plausible Coasean solution, 2 points - failure for large number of firms

4. (4 pts) Suppose that the Coasean solution is not implemented by Alice and Bob. The government decides to correct the market failure by subsidizing the wages of scientists, so Alice and Bob must now pay wage $(1-s)w$. The subsidy is paid for by a lump-sum tax on the earnings of Alice and Bob. What is the optimal wage subsidy rate s?

   With subsidy s, the equilibrium quantity of labor is $l_e = [(1-s)w]^{-2}$. Comparing to the optimal employment level, it is immediate that $s = \frac{1}{2}$ is optimal.

   2 points - solution for equilibrium labor, 2 points - optimal subsidy

5. (4 pts) Now suppose that Alice and Bob are in competition with each other: If Alice improves her widget through R&D, she steals some business from Bob, and this accounts for some of the profits that she earns from innovating. In this case, the profits earned through widget sales are

$$\pi_A(l_A, l_B) = 4(l_A l_B)^{1/4} - wl_A - \sigma l_B \quad \text{and} \quad \pi_B(l_A, l_B) = 4(l_A l_B)^{1/4} - wl_B - \sigma l_A.$$

   Here $\sigma > 0$ parametrizes the strength of "business stealing." Again assuming a positive number of scientists are employed, how many scientists do Alice and Bob employ in equilibrium?

   The equilibrium first order conditions are the same as in part 1, so $l_A = l_B = l_e = (w)^{-2}$.

   1 point - set up profit maximization problem, 1 point - first order condition, 2 points - solution

6. (7 pts) What employment levels are socially optimal? Are equilibrium employment levels too high or too low, and how does this depend on σ? Describe the intuition.

   The interior first order condition for l_A is

$$2l_B^{1/4}l_A^{-3/4} - (w+\sigma) = 0.$$

Solving yields

$$l_A = l_B^{1/3} \left(\frac{w+\sigma}{2} \right)^{-4/3}.$$

The solution for l_B is symmetric, so the (symmetric) optimal employment level satisfies

$$l_* = l_*^{1/3} \left(\frac{w+\sigma}{2} \right)^{-4/3} \iff l_* = \left(\frac{w+\sigma}{2} \right)^{-2}.$$

Equilibrium employment is too low (too high) if and only if $\sigma < (>)\ w$. For intuition, we now have a trade-off between the positive "R&D spillovers" externality and the negative "business stealing" externality. When σ is low, the former dominates, and equilibrium employment is too low. When σ is high, the latter dominates, and equilibrium employment is too high. These effects exactly balance when $\sigma = w$.

2 points - solution, 2 points - high/low answer, 3 points - explanation

Question Three [39 points]

In Massachusetts funding for public universities is being debated. There are two types of people, college students and non-students. College students derive a large benefit from money spent on education. For simplicity assume the entire population of Massachusetts consists of 1 student and 2 non-students. The student is numbered 1, and the non-students 2 and 3. Denote the total spent on education by $E = e_1 + e_2 + e_3$. The utility of the student 1 is

$$U_1 = \ln(x) + \ln(E).$$

Non-students enjoy smaller benefits from higher education spending. Their preferences are

$$U_2 = U_3 = \ln(x) + \frac{1}{2}\ln(E).$$

Both types of people have income Y and spend their money on other goods x or contribute to higher education e. Assume the prices of both are 1.

1. (5pts) What contribution to public education, e_1, will student 1 choose in terms of e_2, e_3 and Y?

   The budget constraint is $Y = x + e_1$, and hence the first order condition for student 1 is:

   $$\frac{1}{e_1 + e_2 + e_3} - \frac{1}{Y - e_1} = 0.$$

   Solving yields

   $$e_1 = \frac{Y - e_2 - e_3}{2}.$$

   3 points for the right FOC, 2 for right solution.

2. (4 pts) What contribution to public education, e_2, will non-student 2 choose in terms of e_1, e_3 and Y? Similarly non-student 2's first order condition is

   $$\frac{1/2}{e_1 + e_2 + e_3} - \frac{1}{Y - e_2} = 0.$$

   Solving yields

   $$e_2 = \frac{Y - 2e_1 - 2e_3}{3}.$$

   2 for the right FOC, 2 for the right solution

3. (4 pts) Explain the effect of an increase of e_1 on e_2. Explain the effect of an increase of e_3 on e_2. Which has a larger impact on e_2 and why?

   They crowd out e_2 equally.This is because only the total expenditure on education matters, not who does the spending. In particular, the marginal utility of one more dollar of expenditure on education for person 2 depends only on the total existing expenditure, not the identity of the spenders.

   1 for saying that e_1 leads to a decrease in e_2, another 1 for e_3 on e_2. 1 for noting the effects are the same and 1 for explaining this is due to only total E mattering.

4. (4 pts) Assuming both non-students contribute the same amount, $e_2 = e_3 = e'_e$, show that in equilibrium $e_1^* = \frac{Y}{2}$ and $e' = 0$. Explain intuitively why $e' = 0$.

   Substituting $e_2 = e_3 = e'$ into the condition from part 1 gives

   $$e_1 = Y/2 - e'.$$

   Substituting this into the conditon from part 2 gives

   $$e' = \frac{Y}{3} - \frac{2}{3}(Y/2 - e') - \frac{2}{3}(e').$$

   Solving yields $e' = 0$, and then we get $e_1 = Y/2$.

   Intuitively, $e' = 0$ because of the free-rider problem and the difference in intensities of preferences for E. The non-students, who don't value education as much as the student, would rather free-ride on the students contribution.

   2 for the right math, 2 for the intuition.

5. The following scheme applies only to this question.

   (4 pts) In question 4 we found that the total expenditure under the private equilibrium to be $E^* = \frac{Y}{2}$. In order to improve fairness, someone suggests we instead fund the same total education equally: everyone pays $\frac{Y}{6}$. Someone else suggests that while $\frac{Y}{6}$ should be the minimum, people should be able to contribute more if they wish to. **Without solving any new optimization problem,** will anyone want to pay more than the $\frac{Y}{6}$ minimum? Explain why or why not in relation to the incentives at the private equilibrium.

   When the total expenditure was E^W, student 1 was indifferent between spending $W/2$ and a tiny bit more. When the total expenditure is E^W student 1 is only spending $E^W/3 = W/6$. Hence they will strictly prefer to spend one more dollar, given the marginal utility of other goods x is lower when $W/6$ is spent on education than when $W/2$ is spent on education by student 1.

   2 for noting the non-students don't want to contribute more, 2 for noting the student does and relating this to the incentives at the private equilibrium in some plausible way.

6. (4 pts) Suppose the government is considering giving a dollar to either student 1 or non-student 2 that had to be spent on education (e.g. a voucher). Which would raise total education spending by more, giving it to student 1 or to non-student 2? Please explain.

   If a dollar was given to the non-student to spend on education, this would raise expenditure by a dollar but this would crowd out some of the student's education spending, specifically 50c of crowd out. If it was given to the student, there would be no crowd-out as the non-students are already spending zero, but the student would treat the dollar as regular income since they were going to spend on education anyway. So the student's education spending would rise by 50c. Hence in this case the impacts are equivalent whether the voucher is given to the student or non-student.

   1 point for realizing the entire dollar must be spent by the non-student, 1 point for crowd out. 1 point for realizing not all of the dollar would be spend by the student, 1 for correct conclusion.

7. (5 pts) If welfare is $W = U_1 + U_2 + U_3$, find the socially optimal level of e_1 and e'. Also find the difference in total education expenditure under the private equilibrium relative to the social optimum.

   Welfare is given by

$$W = \ln(Y - e_1) + 2\ln(Y - e') + 2\ln(e_1 + 2e').$$

   The first order conditions with respect to e_1 and e' are

$$\frac{2}{e_1 + 2e'} - \frac{1}{Y - e_1} = 0,$$
$$\frac{4}{e_1 + 2e'} - \frac{2}{Y - e'} = 0.$$

   This yields

$$\frac{2}{e_1 + 2e'} = \frac{1}{Y_{e1}} = \frac{1}{Y - e'}.$$

   Hence $e' = e_1$ and solving leads to

$$e' = e_1 = \frac{2Y}{5}.$$

   The total education expenditure at the private optimum was $Y/2$ and hence the difference is $6Y/5 - Y/2 = \frac{7Y}{10}$.

8. (5 pts) Suppose the government gives student one a block grant of size $g > 0$. What is the size of the block grant g needed to get total education spending to the social optimum? Is it bigger or smaller than the difference between education spending at the old private equilibrium (from part 4) and the social optimum (from part 7)? Why?

   (Hint: before doing calculus think first about how much the non-students will spend on education)

   Student 1 will spend at weakly more than under the old private equilibrium, and so the non-students will still spend zero (if the marginal utility of a dollar of education spending wasn't worth it before, it can't be now).

   Hence $e_2 = e_3 = 0$. With a grant of size g, student 1 will maximize

$$U_1 = \ln(Y + g - e_1) + \ln(e_1).$$

   Solving the FOC gives

$$e_1 = \frac{Y + g}{2}.$$

   Hence to get total education spending to $6Y/5$ we need that

$$\frac{6Y}{5} = \frac{Y + g}{2} \iff g = \frac{7Y}{5}$$

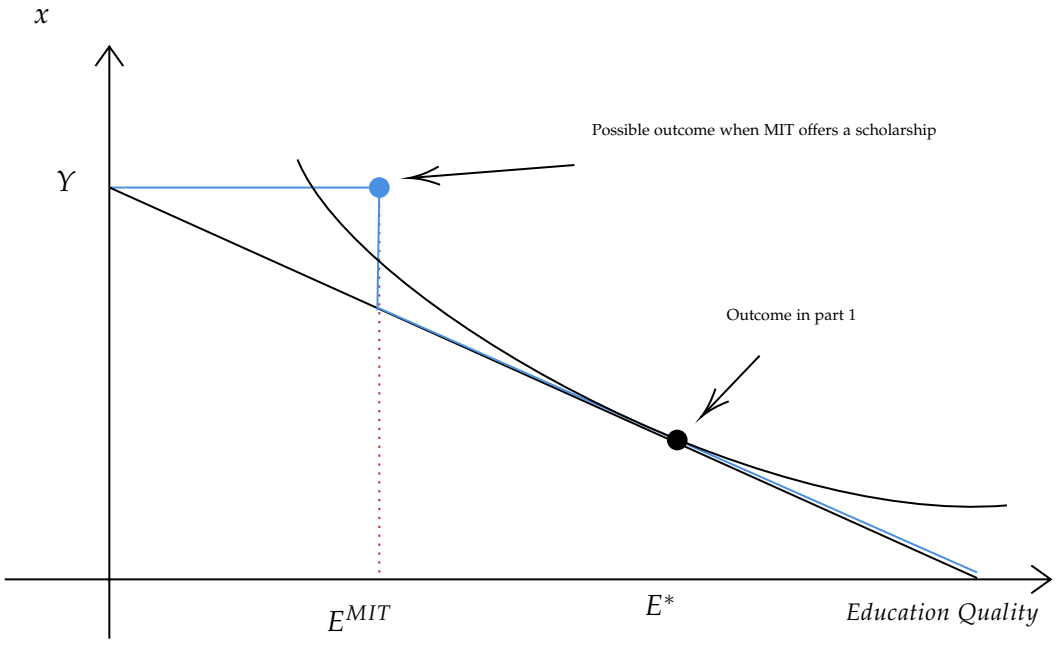
   which is larger than the difference from part 7 which was $\frac{7Y}{10}$ because a block grant will be spent partially on education but also partially on the other goods.

   3 for correct math, 2 for the right intuition.

9. (4 pts) Recall in the private equilibrium level of public education spending was $E^* = \frac{Y}{2}$. Separately, MIT generously decides to offer the student admission for free. Going to MIT delivers utility of $U_1 = \ln(Y) + \ln\left(E^{MIT}\right)$ since the cost is zero all money can be spent on the other good. Assume $E^{MIT} < \frac{Y}{2}$. Assume that if the students attends MIT they do not have to contribute toward the public university.

   Draw the students old (part 1) and new (this part) budget sets, noting their optimal point if they choose MIT and their optimal point if they go to the public university. Explain why financial aid programs at private universities might reduce the total expenditure on education.

   As seen in the diagram, the blue curve is the new budget set, the black line is the old budget set. If the indifference curve tangent to the old equilibrium passes underneath and to the left of the blue point, then the student will go to MIT, spend the rest of their money on x and spend nothing on public education. It is clear (no math needed) that the remaining two non-students won't make up the difference and net public education spending would decline.



1 for correct old budget line, 1 for correct new budget line, 2 for correct explanation .

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