MIT 14.41 – Problem Set 2

Due October 14, 2022
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QUESTION 1: [30 points]

A city needs to decide how much to fund two different public goods: schools and policing. It wants to solicit the preferences of the people in the city to decide on how much of each good to provide.

Suppose that the total level of spending G is fixed at $\bar{G}$ by the city budget, so that all the city government needs to decide on is how to split this total spending between schools and policing. There are a total of 1000 households in the city, and the public goods are funded by an equal tax on each household, so that with spending of $\bar{G}$ each household pays a tax of $t = \frac{\bar{G}}{1000}$.

There are three types of households in the city. These types of households have the same level of income y, but different preferences about public spending. Let X be the fraction of government spending that goes towards schools; the remaining $1 - X$ goes towards policing. (Since X is a fraction, the city cannot choose a value of X outside the range $[0, 1]$.) Households of type A have preferences

$$U_A(y, X, G, t) = \ln(G)\left(1 - (X - 0.1)^2\right) - t + y$$

while households of type B have preferences

$$U_B(y, X, G, t) = \ln(G)\left(1 - (X - 0.4)^2\right) - t + y$$

and households of type C have preferences

$$U_B(y, X, G, t) = \ln(G)\left(1 - (X - 0.8)^2\right) - t + y$$

where $0 < \alpha < 1$. These preferences capture the following properties:

- Households like more spending on both public goods, but get diminishing returns as the spending increases.
- Households dislike having to pay tax for the public goods because it means they get less consumption.
- Households have different preferences about how public goods spending is split between schools and policing, and they get more utility from public goods spending if the split of spending matches their preferences.

There are 200 households of type A, 350 households of type B, and 450 households of type C.

1. (2 points) For each type of household, calculate its ideal choice of X, given that G is fixed at $\bar{G}$. Label these X_A^*, X_B^*, X_C^* respectively.

2. (2 points) Are the preferences of each household single-peaked? Explain why or why not.

   Suppose the city government holds a series of votes between each of these three ideal points. That is, it first holds a vote between X_A^* and X_B^*, then one between X_A^* and X_C^*, then one between X_B^* and X_C^*.

3. (4 points) Identify the majority winner of each of the three votes. Is there a consistent winner (one that beats both of the other two alternatives)? How does this relate to your answer to part 2?

   Now suppose that the government holds a vote over both X and G. It does this in two stages. First, it holds a vote over the level of G. Then it holds votes about the choice of X as described in part 3, taking G as fixed by the first vote.

4. (2 points) What value of X will be chosen in the second stage? Denote this value $\bar{X}$. Note that this will not depend on the level of G.

   Now consider the first-stage vote. Suppose that all the households in the city know that whatever value of G is chosen, the second-stage vote will result in a fraction $\bar{X}$ being spent on schools.

5. (5 points) Calculate the ideal value of G for each type of household, based on your answer to part 4. Label these values G_A^*, G_B^*, G_C^* respectively.

6. (3 points) Suppose the government holds majority votes between G_A^* and G_B^*, G_A^* and G_C^*, and G_B^* and G_C^*. Which outcome will be the consistent winner?

7. (3 points) You should find that the group that gets its ideal choice of X is different from the group that gets its ideal choice of G. Why is this?

   Now, we'll compare the democratic outcome to the outcome if the government knew the voters' preferences and was able to directly choose the utilitarian socially optimal outcome.

8. (1 point) Write down the social welfare function for this economy, which is the sum of each person's utilities.

9. (5 points) The utilitarian government's problem is

$$\max_{G,X} SWF \; s.t. \; G \geq 0, t = \frac{G}{1000}, 0 \leq X \leq 1$$

   i.e. to choose G and X simultaneously to maximise the social welfare function from part 8, subject to the constraints that $G \geq 0$ and $0 \leq X \leq 1$. (Note that once the government chooses G, taxes for each household are fixed at $t = \frac{G}{1000}$ because the budget must be balanced). Calculate the values of G and X that maximise this social welfare function. (Hint: first calculate the optimal value of X)

10. (3 points) Intuitively, why does the utilitarian social optimum (which you calculated in part 9) differ from the democratic outcome (which you calculated in parts 4–6)?

QUESTION 2: [40 points]

Suppose there are 10 people in a neighbourhood. Person 1 has income Y, and the other people all have incomes y, where $y < Y$. The neighbourhood has a public park that everyone in the neighbourhood can use freely. Each person i in the neighbourhood can choose how much of their income they want to spend on a private consumption good, c_i, and how much they want to spend on donations g_i to maintain the park and keep it looking nice. They cannot choose negative amounts for either, so $c_i \geq 0, g_i \geq 0$. Let the total donations for the park be $G = \sum_{i=1}^{10} g_i$; then each person has utility given by

$$U_i = \ln(c_i) + \ln(G)$$

People don't get any additional utility from their spending on the public good (in particular, they have no 'warm glow' utility).

1. (2 points) Calculate person 1's privately optimal choice of g_1 as a function of $g_2, \dots, g_{10}$.
2. (2 points) Calculate person 2's privately optimal choice of g_2 as a function of $g_1, g_3, \dots, g_{10}$.

   Assume that people 2–10 all choose the same level of g_i; denote this level g'.
3. (a) (1 point) If $g' = 0$, what is person 1's privately optimal choice of g_1?

   (b) (5 points) Assuming that $g' > 0$, find the (privately optimal) equilibrium levels of g_1 and g'.
4. (4 points) Based on your answer to part 3b, what inequality in terms of Y and y must be satisfied so that $g' > 0$? What does this imply about how inequality affects free-riding in this context?
5. (4 points) Suppose that $Y = 32, y = 10$. Imagine that the government taxes \$9 away from person 1, and uses this money to give \$1 each to people 2–10. Calculate the new private equilibrium values of g_1 and g'. How does the utility of person 2 change compared to before the tax was introduced? Comment on this result.
6. (4 points) Suppose that the government again taxes \$9 away from person 1, but instead spends that money on the park, so that $G = 9 + g_1 + \cdots + g_{10}$. Calculate the new private equilibrium values of g_1 and g'. How does the utility of person 2 change compared to before the tax was introduced now? Comment on this result.
7. (2 points) Explain how and why your answers to parts 5 and 6 differ.

   Now suppose that person 1 has a private garden which means that they never want to use the public park, and get no utility from it, so person 1's utility is instead given by

   $$U_i = \ln(c_i)$$

   Since they get no utility from the park, they will not want to make any contribution to it.
8. (3 points) Again assuming that in equilibrium players 2–10 choose $g' = g_2 = \cdots = g_{10}$, calculate the new equilibrium value of g' as a function of y, and calculate player 2's utility in equilibrium given that $y = 10$.
9. (4 points) Calculate the utility of player 2 when the government taxes \$9 away from person 1 and gives a dollar to each of the other players.
10. (3 points) Explain how and why your answers to part 5 and part 9 differ.
11. (4 points) Calculate the utility of player 2 when the government taxes \$9 away from person 1 and spends \$9 on the park.
12. (2 points) Comment on your answers to parts 9 and 11.

QUESTION 3: [30 points]

Suppose that the city of Cambridge is planning to remodel and improve all 4 middle schools in the city. Each middle school has 400 students and 50 teachers. Teachers earn on average \$81,000 per year and work on average 50 hours per week for 45 weeks a year (assume there are no distortions and this reflects their per-hour valuation of time outside of work).

The construction will require \$10 million in construction materials per school per year and 1 million hours (total

for all construction workers) of construction labor per school per year. Construction workers earn an equilibrium wage of $20 per hour.

Remodeling each school will take 3 years. Each year, they will start to remodel one new school. Each school will require $0.5 million in maintenance costs per year, starting in the year that it is finished. While each school is being remodeled, the students who would otherwise attend that school are sent to other schools (including outside of Cambridge). This increases transport times for the students by 1 hour per day, and takes away from the time they can spend in extra-curricular activities or doing homework. Assume that each hour outside of school is worth $20 to a student (due to enjoyment, immediate positive effects, and the long-run discounted value of an increase in the likelihood in being admitted to a top college from doing extra-curriculars or more careful homework). Teachers also have to travel to these other schools and travel for an extra hour a day. (You can assume that the construction is isolated on each school campus and doesn't affect anyone else's commute times.) A school year has 180 days.

When finished, each school will have a new library and modern playground, and the school will be a beautiful place to be. This is projected to improve student attendance rates and test scores, and these changes are expected to increase student lifetime earnings by $80,000 each year, starting 15 years after the start of the project. Assume that they remain in Cambridge for the rest of their lives. Finally, higher-income families are expected to move to Cambridge and send their children to the public schools. The city's total income tax revenue is expected to increase by $40M per year, starting in the year when all of the schools are finished. Conditional on the maintenance costs, all of these benefits are expected to go on forever.

Assume that the private-market alternative to funding this project would be a financial investment that returned 8% per year. Assume the income tax is a flat 5% tax.

1. (3 points) What are three ways that a policy analyst could have come up with the value of middle-schoolers' time? Explain how each would work in the context of this example.

2. (6 points) **Economic costs:** Calculate each of the economic costs associated with the project. Then compute the total cost of the project. *Throughout, feel free to round to the nearest million dollars.*

3. **Economic benefits:**
   (a) (4 points) Calculate each of the economic benefits associated with the project for *the city of Cambridge*. Then compute the total benefit of the project to the city.
   (b) (4 points) Now assume you work for the federal Department of Education and are deciding whether to provide a grant to the city of Cambridge, and they have proposed to use it to remodel their middle schools. Calculate each of the economic benefits associated with the project for *society as a whole*. Then compute the total benefit of the project to society.
   (c) (2 points) Would the city of Cambridge want to embark on this project? Should the federal government give them a grant to do so? Why or why not?

4. (4 points) Now, imagine an ordinance passed and the city must pay all contractors at least $30 per hour. How does this change the cost-benefit analysis, and why?

5. (3 points) Imagine that the same exact project (with the same exact costs and benefits) was proposed in 4 middle schools in the Boston Public School system. Average household incomes of public school students in Boston are much lower than in Cambridge. Why might the federal DOE decide to give the grant to Boston Public Schools instead of Cambridge Public Schools?

6. (4 points) The DOE decided to provide a fixed $335M grant to Boston Public Schools on the condition that they use it to remodel their four oldest middle schools.
   (a) (2 points) What type of grant is the DOE providing to BPS?
   (b) (2 points) Why do you think the DOE is providing this grant instead of some other type of transfer?

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