MIT 14.41 – Problem Set 3

Due October 21, 2022
Submit **online** by 5pm ET using Gradescope

QUESTION 1: Education [38 points]

Consider a society with four types of families. Each family chooses s, an amount of quality schooling, and c, all other consumption. Assume that c is a numeraire good with price 1 and the price of one more unit of school quality is $p = \frac{1}{4}$. The four types of families are as follows:

$$y_a = 100; u_a = ln(c) + 4ln(s)$$

$$y_b = 100; u_b = ln(c) + ln(s)$$

$$y_c = 40; u_c = ln(c) + ln(s)$$

$$y_d = 40; u_d = ln(c) + 4ln(s)$$

There are 400 families of type A, 100 of type B, 200 of type C, and 300 of type D.

Assume that the future income of a child is $y^1 = \frac{y^0}{2} + \left(\frac{s}{4}\right)^{\frac{3}{2}}$, where y^0 is their household income when they are a child.

1. (2 points) Do parents' preferences take children's earnings fully into account when they choose their children's level of schooling? How can you tell?

2. (4 points) First, let's solve for each family's decision when there is no public schooling. Find each type of family's chosen c, s.

3. Now, assume the government introduces public schools which provide the same quality schooling as choosing $s = E$, for free. If a family chooses to send their child to public school, they must consume exactly E education.
   - (a) (2 points) Under what conditions on E will each type of family choose to send their child to public school?
   - (b) Suppose the government set $E = 45$
     - i. (2 points) Which families send their children to public school?
     - ii. (2 points) What does this policy do to total productivity (measured by total future income of children)?
     - iii. (2 points) What happens to inequality in this society? Here and for the rest of the problem, think about inequality in terms of the range of **children's future incomes**.
   - (c) Now instead suppose the government set $E = 150$
     - i. (2 points) Which families send their children to public school?
     - ii. (4 points) What does this policy do to total productivity (measured by total children's future income), relative to when there were no public schools?

iii. (2 points) What happens to inequality in this society?

(d) (4 points) Discuss how and why parts (b) and (c) of this question differ in terms of productivity and inequality.

4. Instead of introducing public schools, suppose that in addition to their income, each family gets voucher of value V dollars from the government that they can use on education.

   (a) (4 points) Write down each family's utility maximization problem, and solve for each family's choice of s and c (given V).

   (b) (4 points) What level of schooling would each family choose if $V = 11.25$ *dollars*? How would this affect productivity and inequality, relative to the world without any public education or voucher system? (Note that, in terms of units of schooling, the voucher is worth $4 \times 11.25 = 45$ units)

5. Now, let's think about the differences you found in the effects of the policies in part 2 and part 3.

   (a) (2 points) If the government is politically or financially constrained and can only offer $E = 45$ or $V = 11.25$ (worth 45 units of schooling), which should they do? Comment on the different effects of these policies on productivity and inequality.

   (b) (2 points) If the government is unconstrained, should they offer the the policy you recommended in (a) or $E = 150$? Comment on the different effects of these policies on productivity and inequality.

QUESTION 2: Selection in Health Insurance Markets [27 points]

Suppose there are three types of people in the world:

- Type A: People at high risk of having serious complications from Covid-19, who have a large probability $p_h \in (0, 1)$ of being hospitalized due to Covid in a given month, who have standard levels of risk aversion
- Type B: People at low risk of having serious complications, who have a smaller probability $p_l \in (0, p_h)$ of being hospitalized, who have standard levels of risk aversion
- Type C: People at low risk of having serious complications, who also have the smaller probability p_l of being hospitalized, who are more risk averse than the rest of the population.

Everyone has utility of the form:

$$u = \frac{c^{1-\eta}}{1-\eta}$$

The various levels of risk aversion are represented by types A and B having $\eta = 0.55$ and type C having $\eta = 0.75$

A fraction α of people are type A, a fraction β are type B, and the remainder are type C. Assume that everyone has an income stream of $2w$ and health care costs are w for anyone hospitalized with Covid.

1. (5 points) Initially suppose that competitive health insurers offer insurance contracts of the following form: If a person pays $m \geq 0$ in premiums, they receive a payout mb when they are hospitalized due to Covid. And, insurers can observe what type of person one is, so they can offer different contracts to each type. What contracts b_A, b_B, and b_C arise in equilibrium for each type of person, and what premiums m_A, m_B, and m_C are chosen by each type? (*Hint*: In a competitive equlibrium in an insurance market, insurers make zero profits).

2. (2 points) In this market, what is the effect on premiums of a government policy that mandates full insurance (if a person pays some amount m in premiums charged by the insurance company, their hospital bills are paid completely by the insurer).

3. (5 points) Next, we will study a setting where full insurance continues to be mandated, but insurers can only offer one type of contract to all people. First, what is the highest premium at which each type of person would buy insurance? Rank the types in terms of their demand for insurance. Under what condition will people of type C demand more insurance than those of type A?

4. When insurers offer a single premium m, three outcomes are possible: a *pooling equilibrium* where all three types of people purchase insurance, a *two-buyer separating equilibrium* in which two types purchase insurance, or a *one-buyer separating equilibrium* in which one type purchases insurance. In either case, equilibrium requires that insurance firms earn zero profits.

   (a) (3 points) Let's start with a pooling equilibrium. Solve for the premium at which an insurer would be willing to offer a contract in a pooling equilibrium. Under what condition(s) are all three types willing to pay this price?

   (b) (4 points) Now, let's consider a two-buyer separating equilibrium. Which two types will purchase insurance in this type of equilibrium? Solve for the premium at which an insurer would be willing to offer just those two types a contract. Under what condition(s) are only these two types willing to buy insurance?

   (c) (3 points) Finally, let's consider a one-buyer separating equilibrium. Which type will purchase insurance in this type of equilibrium? Solve for the premium at which an insurer would be willing to offer only this type a contract. Under what condition(s) is only this one type willing to buy insurance?

5. (3 points) Compare the equilibria in parts 3 and 4 (without type-specific contracts) to the equilibrium in part 2 (with type-specific contracts). Who is better-off, and who is worse-off, in parts 3 and 4 relative to part 2?

6. (2 points) Based on this example, what can you say about how variation in risk aversion affects the feasibility of a private market for health insurance? What is the intuition behind this?

QUESTION 3: Moral Hazard [35 points]

Ken is at risk of contracting a disease. With probability π, he will end up getting sick, and with probability $1 - \pi$ he will stay healthy. If he contracts the disease, treating him will cost \$$c$. Assume that the financial cost is the only cost of the disease, so that he does not lose any additional utility from the disease. His income is $y > c$. Without insurance, his expected utility is therefore given by

$$E[u] = \pi u(y - c) + (1 - \pi)u(y)$$

Now suppose that he has access to insurance. The insurance company will cover the full cost of Ken's healthcare if Ken gets sick. The company charges Ken a premium p for this insurance product, which he has to pay whether he gets sick or not. Thus if he buys insurance at price p, his expected utility is now

$$E[u] = \pi u(y - p) + (1 - \pi)u(y - p) = u(y - p)$$

Throughout this question, the 'actuarially fair' price refers to the premium that is equal to the expected costs that the insurance company has to pay; if the insurance company chooses this premium, it will make zero profit in expectation. Also throughout this question, you can assume that if Ken gets exactly the same expected utility with and without insurance, he will choose to buy insurance.

1. (2 points) What is the actuarially fair premium p for this insurance product? Write down a condition in terms of c and π, and then calculate the value of the premium when $c = 500, \pi = 0.2$.

2. (2 points) Write down a condition on p that must be satisfied for Ken to be indifferent between buying full insurance and not buying insurance, in terms of $u(\cdot), y, \pi, p$, and c.

   Suppose that $y = 1000, c = 500, \pi = 0.2$. Suppose that Ken's utility function is

$$u(x) = \begin{cases} \ln(x) & \text{if } \sigma = 1 \\ \frac{1}{1-\sigma}x^{1-\sigma} & \text{if } \sigma \neq 1 \end{cases}$$

   This means that σ is Ken's *coefficient of relative risk aversion*: higher σ means that he dislikes risk more. Note that $\ln(x)$ is the limit of $\frac{1}{1-\sigma}x^{1-\sigma}$ as $\sigma \to 1$. Throughout this question, when σ is not specified you should assume that $\sigma \geq 0$.

3. (6 points) What is the maximum amount that Ken will be willing to pay for full insurance:
   (a) When $\sigma = 0$?
   (b) When $\sigma = 1$?
   (c) When $\sigma = 2$?

   Now suppose that Ken can reduce his chances of contracting the disease by taking a new vaccine. The vaccine costs him \$$v$, and reduces his chances of getting the disease to $\frac{\pi}{2}$. It has no other direct effect on his utility (for instance, he does not get disutility from having to get an injection).

4. (2 points) First, imagine there is no insurance available. Write down the condition for the price v that must be satisfied for Ken to be indifferent between getting the vaccine and not getting the vaccine, in terms of $u(.), y, \pi, c$, and v.

5. (2 points) Again suppose that $y = 1000, c = 500, \pi = 0.2$. What is the maximum amount that Ken would be willing to pay for the vaccine when $\sigma = 0$?

6. (4 points) Suppose that the vaccine is actually available for \$20, and that insurance does not cover the cost of the vaccine, so that Ken has to pay \$20 for the vaccine even when he has insurance coverage. Why is there a moral hazard problem when Ken has full insurance? Does this moral hazard problem depend on Ken's value of σ? Note that $\sigma \geq 0$, and that when $\sigma > 0$, Ken's maximum willingness to pay for the vaccine when he doesn't have insurance will always be strictly greater than the amount you calculated in part 5.

7. (6 points) Once again assume that $\pi = 0.2, c = 500, y = 1000$, and now assume $v = 20$. Taking into account the fact that Ken can choose to buy the vaccine when he does not have insurance, how much would he be willing to pay for full insurance when $\sigma = 1$? Comment on your answer and why it differs from your answer to part 3b.

8. (3 points) Another insurance company offers a different insurance product; this product still covers the full cost of Ken's healthcare if he gets sick, but it also covers the cost of the vaccine. Assume that Ken will definitely (with 100% probability) get the vaccine when he doesn't have to pay extra for it. What would the actuarially fair premium for this insurance product be? Write an expression for p in terms of π, c, and v, and then calculate the value of p when $\pi = 0.2, c = 500, v = 20$.

9. (4 points) Ken now has three options: an insurance product that covers the cost of the vaccine, an insurance product that does not cover the cost of the vaccine, and not buying insurance (but possibly buying the vaccine without insurance). Assume both insurance products charge the actuarially fair premiums. Rank these three options in order of how much utility Ken gets from each when $\sigma = 1$. (Hint: you do not need to calculate the level of utility; use your answers to parts 1, 7, and 8.)

10. (4 points) Suppose now that there is no vaccine for this disease, but Ken is instead able to reduce his risk of getting the disease by eating a restricted diet. Assume he would dislike this restricted diet compared to his ideal diet. Why might it be harder for the insurance market to address this form of moral hazard? (No math required for this part.)