MIT 14.41 – Problem Set 2

Due October 14, 2022
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QUESTION 1: [30 points]

A city needs to decide how much to fund two different public goods: schools and policing. It wants to solicit the preferences of the people in the city to decide on how much of each good to provide.

Suppose that the total level of spending G is fixed at $\bar{G}$ by the city budget, so that all the city government needs to decide on is how to split this total spending between schools and policing. There are a total of 1000 households in the city, and the public goods are funded by an equal tax on each household, so that with spending of $\bar{G}$ each household pays a tax of $t = \frac{\bar{G}}{1000}$.

There are three types of households in the city. These types of households have the same level of income y, but different preferences about public spending. Let X be the fraction of government spending that goes towards schools; the remaining $1 - X$ goes towards policing. (Since X is a fraction, the city cannot choose a value of X outside the range $[0, 1]$.) Households of type A have preferences

$$U_A(y, X, G, t) = \ln(G)\left(1 - (X - 0.1)^2\right) - t + y$$

while households of type B have preferences

$$U_B(y, X, G, t) = \ln(G)\left(1 - (X - 0.4)^2\right) - t + y$$

and households of type C have preferences

$$U_B(y, X, G, t) = \ln(G)\left(1 - (X - 0.8)^2\right) - t + y$$

where $0 < \alpha < 1$. These preferences capture the following properties:

- Households like more spending on both public goods, but get diminishing returns as the spending increases.
- Households dislike having to pay tax for the public goods because it means they get less consumption.
- Households have different preferences about how public goods spending is split between schools and policing, and they get more utility from public goods spending if the split of spending matches their preferences.

There are 200 households of type A, 350 households of type B, and 450 households of type C.

1. (2 points) For each type of household, calculate its ideal choice of X, given that G is fixed at $\bar{G}$. Label these X_A^*, X_B^*, X_C^* respectively.

Solution: $X_A^* = 0.1, X_B^* = 0.4, X_C^* = 0.8$

Grading notes: 1 point for setting up FOCs correctly but with some mathematical mistakes. Full credit if they just identify the correct values from the functional form without setting up the maximisation problem.

2. (2 points) Are the preferences of each household single-peaked? Explain why or why not.

Solution:

Yes: quadratic loss function in X.

Grading notes: People can make the argument either intuitively by considering the functional form, or my looking for concavity of the function / some other mathematical condition that guarantees single-peakedness. No points for the wrong answer.

Suppose the city government holds a series of votes between each of these three ideal points. That is, it first holds a vote between X_A^* and X_B^*, then one between X_A^* and X_C^*, then one between X_B^* and X_C^*.

3. (4 points) Identify the majority winner of each of the three votes. Is there a consistent winner (one that beats both of the other two alternatives)? How does this relate to your answer to part 2?

Solution: 0.4 beats 0.1 and 0.8, 0.1 beats 0.8, so 0.4 is the Condorcet winner. Single-peaked preferences ensure that there is a consistent winner at the median by Median Voter Theorem.

Grading notes: Up to three points for correctly identifying the winners of each of the votes and the Condorcet winner; one point for realising that single-peaked preferences imply that the MVT holds.

Now suppose that the government holds a vote over both X and G. It does this in two stages. First, it holds a vote over the level of G. Then it holds votes about the choice of X as described in part 3, taking G as fixed by the first vote.

4. (2 points) What value of X will be chosen in the second stage? Denote this value $\bar{X}$. Note that this will not depend on the level of G.

Solution: $\bar{X} = 0.4$ as argued above.

Grading notes: Full credit for an answer that's incorrect but consistent with the answer to part 3.

Now consider the first-stage vote. Suppose that all the households in the city know that whatever value of G is chosen, the second-stage vote will result in a fraction $\bar{X}$ being spent on schools.

5. (5 points) Calculate the ideal value of G for each type of household, based on your answer to part 4. Label these values G_A^*, G_B^*, G_C^* respectively.

Solution: G_A^* maximises $\ln(G)\left(1-(0.1-0.4)^2\right)-\frac{G}{1000} \iff \frac{1}{G}\left(1-(0.1-0.4)^2\right)-\frac{1}{1000}=0 \iff G=1000(0.91)=910$. Similar calculations for B and C give $G_B^*=1000, G_C^*=840$.

Grading notes: Full credit for an answer that is correct except for being based on plugging in an incorrect answer to part 4. 3 points for setting up the FOCs correctly but making mathematical mistakes in solving for G^*.

6. (3 points) Suppose the government holds majority votes between G_A^* and G_B^*, G_A^* and G_C^*, and G_B^* and G_C^*. Which outcome will be the consistent winner?

Solution: $G_A^*=910$ is the Condorcet winner (can apply median voter theorem or look for the winner of each pairwise vote).

Grading notes: 1 point for answer that uses right process (median voter theorem or looking at winner of pairwise votes) but gets the winner wrong.

7. (3 points) You should find that the group that gets its ideal choice of X is different from the group that gets its ideal choice of G. Why is this?

Solution: B is the median voter in the second stage, but that means they get their ideal split of public goods, so they get the most value from each unit of public goods spending. This means they must be on the extreme in the vote over G as they want more public goods spending than either of the other groups. Thus they aren't the median voter over G and so don't get their ideal choice of G; instead the group that is closest to them, group A, gets its ideal.

Grading notes: 2 point for observing that the median voter over G is different from the median voter over A, 1 point for relating this to the fact that B gets its ideal X which means it gets the most value from government spending and thus is not the median voter over G.

Now, we'll compare the democratic outcome to the outcome if the government knew the voters' preferences and was able to directly choose the utilitarian socially optimal outcome.

8. (1 point) Write down the social welfare function for this economy, which is the sum of each person's utilities.

Solution:

$$\begin{aligned} SWF(G,X) =& 200\left[\ln(G)\left(1-(X-0.1)^2\right)-\frac{G}{1000}+y\right] \\ &+350\left[\ln(G)\left(1-(X-0.4)^2\right)-\frac{G}{1000}+y\right] \\ &+450\left[\ln(G)\left(1-(X-0.8)^2\right)-\frac{G}{1000}+y\right] \\ =& \ln(G)\left(1000-200(X-0.1)^2-350(X-0.4)^2-450(X-0.8)^2\right)-G+1000y \end{aligned}$$

Grading notes: Make sure to give the point for any equivalent rearrangements of this SWF.

9. (5 points) The utilitarian government's problem is

$$\max_{G,X} SWF \text{ s.t. } G \geq 0, t = \frac{G}{1000}, 0 \leq X \leq 1$$

i.e. to choose G and X simultaneously to maximise the social welfare function from part 8, subject to the constraints that $G \geq 0$ and $0 \leq X \leq 1$. (Note that once the government chooses G, taxes for each household are fixed at $t = \frac{G}{1000}$ because the budget must be balanced). Calculate the values of G and X that maximise this social welfare function. (Hint: first calculate the optimal value of X)

Solution: Differentiating SWF with regard to X gives

$$-400(X-0.1) - 700(X-0.4) - 900(X-0.8) = 0 \iff X^* = 0.52$$

Substituting this into the SWF gives us

$$\begin{aligned} SWF(G, X^*) &= \ln(G)\left(1000 - 200(0.52-0.1)^2 - 350(0.52-0.4)^2 - 450(0.52-0.8)^2\right) - G + 1000y \\ &= 924.4\ln(G) - G + 1000y \end{aligned}$$

which implies $G^* = 924.4$.

Grading notes: 2 points for the right process (calculate X, substitute back into SWF and calculate optimal G) even if there are arithmetic mistakes. 3 points for having the right FOCs. If X^* is wrong but G^* is right given X^*, don't remove additional points.

10. (3 points) Intuitively, why does the utilitarian social optimum (which you calculated in part 9) differ from the democratic outcome (which you calculated in parts 4–6)?

Solution: The utilitarian social optimum is able to take into account the strength of preferences, as well as preferences of people away from the median. There is a large group of people who would like substantially higher X (group C), but because the group is smaller than the median, their preferences are not represented democratically whereas they are represented by the utilitarian government.

Grading notes: 1 point for making a point about strength / intensity of preferences, 1 for observing that in this setting X is higher because group C is large and has a strong preference for higher X, 1 for making some argument about what this implies for G.

QUESTION 2: [40 points]

Suppose there are 10 people in a neighbourhood. Person 1 has income Y, and the other people all have incomes y, where $y < Y$. The neighbourhood has a public park that everyone in the neighbourhood can use freely. Each person i in the neighbourhood can choose how much of their income they want to spend on a private consumption good, c_i, and how much they want to spend on donations g_i to maintain the park and keep it looking nice. They cannot choose negative amounts for either, so $c_i \geq 0, g_i \geq 0$. Let the total donations for the park be $G = \sum_{i=1}^{10} g_i$; then each person has utility given by

$$U_i = \ln(c_i) + \ln(G)$$

People don't get any additional utility from their spending on the public good (in particular, they have no 'warm glow' utility).

1. (2 points) Calculate person 1's privately optimal choice of g_1 as a function of $g_2, \cdots, g_{10}$.

> **Solution:** 1's utility is $ln(Y - g_1) + \ln(g_1 + g_2 + \cdots + g_{10})$, which implies the FOC
>
> $$\frac{1}{g_1 + \cdots + g_{10}} = \frac{1}{Y - g_1}$$
>
> Rearranging this gives
>
> $$g_1 = \frac{1}{2}(Y - \sum_{i=2}^{10} g_i)$$
>
> **Grading notes:** 1 point for the right FOC, 1 for correctly solving for g_1.

2. (2 points) Calculate person 2's privately optimal choice of g_2 as a function of $g_1, g_3, \cdots, g_{10}$.

> **Solution:** Similarly, 2's utility is $ln(y - g_2) + \ln(g_1 + g_2 + \cdots + g_{10})$, which implies the FOC
>
> $$\frac{1}{g_1 + g_2 \cdots + g_{10}} = \frac{1}{y - g_2}$$
>
> Rearranging this gives
>
> $$g_2 = \frac{1}{2}(y - g_1 - \sum_{i=3}^{10} g_i)$$
>
> **Grading notes:** 1 point for the right FOC, 1 for correctly solving for g_1.

Assume that people 2–10 all choose the same level of g_i; denote this level g'.

3. (a) (1 point) If $g' = 0$, what is person 1's privately optimal choice of g_1?

   > **Solution:** If $g' = 0$ then $G = g_1$; substituting this into the expression from part 1 gives $g_1 = \frac{1}{2}Y$ **Grading notes**: full credit for an incorrect answer that is consistent with their answer to part 1.

   (b) (5 points) Assuming that $g' > 0$, find the (privately optimal) equilibrium levels of g_1 and g'.

   > **Solution:** $g_1 = \frac{1}{2}(Y - 9g'), g' = \frac{1}{2}(y - g_1 - 8g') \iff 5g' = \frac{1}{2}(y - g_1) \iff g' = \frac{1}{10}(y - g_1)$. Solving this system of two equations gives us: $g_1 = \frac{10}{11}Y - \frac{9}{11}y, g' = \frac{2}{11}y - \frac{1}{11}Y$

   Grading notes: 1 point for each equation set up correctly, up to 3 points for correctly solving the equations.

4. (4 points) Based on your answer to part 3b, what inequality in terms of Y and y must be satisfied so that $g' > 0$? What does this imply about how inequality affects free-riding in this context?

Solution: $g' > 0$ when $Y < 2y$, so free-riding is more extreme when inequality is higher.

Grading notes: 2 points for the correct inequality, full credit for an answer that is correct given the answer to 3b. 2 points for observing this means that inequality leads to more free-riding because people are more likely to choose $g' = 0$.

5. (4 points) Suppose that $Y = 32, y = 10$. Imagine that the government taxes \$9 away from person 1, and uses this money to give \$1 each to people 2–10. Calculate the new private equilibrium values of g_1 and g'. How does the utility of person 2 change compared to before the tax was introduced? Comment on this result.

Solution: 2...10 contribute 0 both before and after, because the inequality holds. 1 contributes 16 before, 11.5 after due to the fall in income. 2 gains utility of $\ln(11) - \ln(10) \approx 0.09$ from higher consumption, but loses utility of $\ln(16) - \ln(11.5) \approx 0.33$ from lower public good, so worse off overall.

Grading notes: 1 point for correctly concluding that people 2-10 contribute 0 before and after, 2 point for correctly calculating the change in 1's public good contribution, 1 point for plugging these numbers into the utility function correctly.

6. (4 points) Suppose that the government again taxes \$9 away from person 1, but instead spends that money on the park, so that $G = 9 + g_1 + \cdots + g_{10}$. Calculate the new private equilibrium values of g_1 and g'. How does the utility of person 2 change compared to before the tax was introduced now? Comment on this result.

Solution: 2...10 contribute 0 before and after (the condition for player 2 to contribute 0 is now $2y > Y + 9$, so it becomes less desirable for players 2...10 to contribute). 1 contributes 16 before, 7 after (full crowd-out; we can show this from the optimisation problem of player 1 or argue it intuitively). No change in utility for player 2, as total public goods spending doesn't change and private consumption doesn't change.

Grading notes: 1 point for correctly concluding that people 2-10 contribute 0 before and after, 2 points for identifying that there will be full crowd-out of player 1's contributions, 1 point for concluding no change in utility.

7. (2 points) Explain how and why your answers to parts 5 and 6 differ.

Solution: When tax revenue is split between the other people in society, it gives them a small increase in consumption, but leads to a large decrease in the public good because they were free-riding off the rich person's contributions; since the public good is under-provided to begin with, reducing the level of the public good makes everyone worse off. When it is spent on the park, this offsets the decrease in the private person's contribution so it does not make them worse off, but it does not make them better off either because the private contributions are fully crowded out.

Grading notes: 1 point for identifying that the increase in consumption is outweighed by the decrease in the public good in part 5, 1 point for identifying that government spending on the public good in part 6 offsets this to make it neutral.

Now suppose that person 1 has a private garden which means that they never want to use the public park, and get no utility from it, so person 1's utility is instead given by

$$U_i = \ln(c_i)$$

Since they get no utility from the park, they will not want to make any contribution to it.

8. (3 points) Again assuming that in equilibrium players 2–10 choose $g' = g_2 = \cdots = g_{10}$, calculate the new equilibrium value of g' as a function of y, and calculate player 2's utility in equilibrium given that $y = 10$.

> **Solution:**
> $$g' = \frac{1}{2}(y - 8g'), 5g' = \frac{1}{2}y, g' = \frac{1}{10}y.$$
> Utility is $\ln(9) + \ln(9) \approx 4.39$.
>
> **Grading notes:** 1 point for right FOC, 1 point for right equilibrium value of g', 1 point for right utility value.

9. (4 points) Calculate the utility of player 2 when the government taxes \$9 away from person 1 and gives a dollar to each of the other players.

> **Solution:** Transfer of \$1: each person gets income of \$11, spends \$9.9 on private consumption and \$1.1 on the public good for total public goods spending of \$9.9; increase of $\ln(9.9) - \ln(9)$ from the transfer and increase of $\ln(9.9) - \ln(9)$ from higher public goods spending.
>
> **Grading notes:** Up to 3 points for calculation of the new optimal consumption and public goods spending, 1 point for correctly plugging these values into the utility function.

10. (3 points) Explain how and why your answers to part 5 and part 9 differ.

> **Solution:** In part 5 redistribution to player 2 makes them worse off, but in part 9 it makes them better off. This is because in part 9 a non-contributor is made to contribute to the public good, which makes the people who benefit from it better off, whereas in part 5 the tax crowds out contributions from the person who was funding the public good and the utility lost from this exceeds the utility gained from higher consumption.
>
> **Grading notes**: Key point is that in part 5 a contributor to the public good is being taxed, while in part 9 a non-contributor is taxed, with different implications for crowd out.

11. (4 points) Calculate the utility of player 2 when the government taxes \$9 away from person 1 and spends \$9 on the park.

> **Solution:** \$9 extra on park: new equilibrium choice is $g' = \frac{1}{10}(y-9) = \frac{1}{10}$. Total public goods spending goes up to 9.9 as each person contributes 0.1 (so utility increases by $\ln(9.9) - \ln(9)$), and consumption increases to 9.9 (so utility increases again by $\ln(9.9) - \ln(9)$. Total utility is thus the same as in part 9.
>
> **Grading notes**: Up to 3 points for calculation of the new optimal consumption and public goods spending, 1 point for correctly plugging these values into the utility function.

12. (2 points) Comment on your answers to parts 9 and 11.

Solution: In this case utility is the same whether money is given to the people directly or spent on the public good, and in both cases it increases relative to the laissez-faire outcome. The public good is under-provided in both cases, but the government funding the public good doesn't improve the outcome because it leads to partial crowd-out from the people funding it.

Grading notes: Potential points for observing that utility and public goods provision are the same either way, that utility is higher than the laissez-faire outcome, and that government provision of the public good in part 11 doesn't undo the under-provision in part 9 due to crowd-out.

QUESTION 3: [30 points]

Suppose that the city of Cambridge is planning to remodel and improve all 4 middle schools in the city. Each middle school has 400 students and 50 teachers. Teachers earn on average $81,000 per year and work on average 50 hours per week for 45 weeks a year (assume there are no distortions and this reflects their per-hour valuation of time outside of work).

The construction will require $10 million in construction materials per school per year and 1 million hours (total for all construction workers) of construction labor per school per year. Construction workers earn an equilibrium wage of $20 per hour.

Remodeling each school will take 3 years. Each year, they will start to remodel one new school. Each school will require $0.5 million in maintenance costs per year, starting in the year that it is finished. While each school is being remodeled, the students who would otherwise attend that school are sent to other schools (including outside of Cambridge). This increases transport times for the students by 1 hour per day, and takes away from the time they can spend in extra-curricular activities or doing homework. Assume that each hour outside of school is worth $20 to a student (due to enjoyment, immediate positive effects, and the long-run discounted value of an increase in the likelihood in being admitted to a top college from doing extra-curriculars or more careful homework). Teachers also have to travel to these other schools and travel for an extra hour a day. (You can assume that the construction is isolated on each school campus and doesn't affect anyone else's commute times.) A school year has 180 days.

When finished, each school will have a new library and modern playground, and the school will be a beautiful place to be. This is projected to improve student attendance rates and test scores, and these changes are expected to increase student lifetime earnings by $80,000 each year, starting 15 years after the start of the project. Assume that they remain in Cambridge for the rest of their lives. Finally, higher-income families are expected to move to Cambridge and send their children to the public schools. The city's total income tax revenue is expected to increase by $40M per year, starting in the year when all of the schools are finished. Conditional on the maintenance costs, all of these benefits are expected to go on forever.

Assume that the private-market alternative to funding this project would be a financial investment that returned 8% per year. Assume the income tax is a flat 5% tax.

1. (3 points) What are three ways that a policy analyst could have come up with the value of middle-schoolers' time? Explain how each would work in the context of this example.

Solution: You could use market-based measures, for example, average wages for 14 year-olds in states where they are allowed to work. You could ask them (contingent valuation): for example, you could survey middle schoolers and ask them how much you would have to pay them to sit on a bus for an extra hour each day. You could also try to measure it through revealed preference: by measuring how school enrollment or house prices respond to longer bus routes (assuming that you could isolate longer bus routes as the only characteristic that is different between two sets of schools).
Grading notes: 1 point per method + a context-appropriate example. (0.25 points per method for just mentioning the name (market-based, contingent valuation, revealed preference). If explanations are correct, don't need to use the name of each.

2. (6 points) **Economic costs:** Calculate each of the economic costs associated with the project. Then compute the total cost of the project. *Throughout, feel free to round to the nearest million dollars.*

Solution: Note: in year 0 there is 1 school under construction, in year 1 there are 2 schools, in year 2 there are 3 schools, in year 3 there are 3 schools, in year 4 there are 2 schools, and in year 5 there is 1 school under construction. And, throughout, $r = 0.08$.

- Construction materials: \$10M per year per school under construction

$$\sum_{t=0}^{t=2} \frac{\$10M}{(1+r)^t} + \sum_{t=1}^{t=3} \frac{\$10M}{(1+r)^t} + \sum_{t=2}^{t=4} \frac{\$10M}{(1+r)^t} + \sum_{t=3}^{t=5} \frac{\$10M}{(1+r)^t}$$

$$= \frac{\$10M}{(1+r)^0} + 2 * \frac{\$10M}{(1+r)^1} + 3 * \frac{\$10M}{(1+r)^2} + 3 * \frac{\$10M}{(1+r)^3} + 2 * \frac{\$10M}{(1+r)^4} + \frac{\$10M}{(1+r)^5}$$

$$= \$100M$$

- Construction labor costs: \$20M (equilibrium wage*number of workers) per year per school under construction. So, plug in \$20M wherever there is \$10M above: $= \$200M$

- Maintenance costs: \$0.5M per year per school once it is finished. One school is done in year 3, two in year 4, etc. until all are done in year 6 and require maintenance forever

$$\frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \sum_{t=6}^{\infty} 4 * \frac{\$0.5M}{(1+r)^t}$$

$$= \frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \frac{1}{(1+r)^6} \sum_{t=0}^{\infty} 4 * \frac{\$0.5M}{(1+r)^t}$$

$$= \frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \frac{1}{(1+r)^5} * 4 * \frac{\$0.5M}{r}$$

$$= \$19M$$

- Teacher travel costs: Teachers earn $81000/(50*45) = \$36$ per hour, which we will use as their valuation of time spent (working and not working). Per year, they lose 180 hours when their school is under construction. With 50 teachers per school, that's \$324,000 per school per year. Altogether:

$$= \frac{324,000}{(1+r)^0} + 2 * \frac{324,000}{(1+r)^1} + 3 * \frac{324,000}{(1+r)^2} + 3 * \frac{324,000}{(1+r)^3} + 2 * \frac{324,000}{(1+r)^4} + \frac{324,000}{(1+r)^5}$$

$$= \$3M$$

- Student travel costs: 400 students per school, 180 hours per student per year, $20 per student per hour is $1,440,000 per school per year. Altogether:

$$= \frac{1,440,000}{(1+r)^0} + 2 * \frac{1,440,000}{(1+r)^1} + 3 * \frac{1,440,000}{(1+r)^2} + 3 * \frac{1,440,000}{(1+r)^3} + 2 * \frac{1,440,000}{(1+r)^4} + \frac{1,440,000}{(1+r)^5}$$

$$= \$14M$$

Summing it up, the total economic cost of the project is:

$$\$100M + \$200M + \$19M + \$3M + \$14M = \$336M$$

Grading notes: 1 point for construction materials, 1 point for labor costs, 1 point for maintenance costs, 1 point for teacher and travel costs, 1 point for student travel costs, 1 point for summing it up. (2 points of extra credit if they mention anything about why we might want to use a different valuation of teacher time).

3. **Economic benefits:**

(a) (4 points) Calculate each of the economic benefits associated with the project for *the city of Cambridge*. Then compute the total benefit of the project to the city.

Solution:

- Tax revenue from student lifetime earnings: $4,000 per kid per year, and 1600 kids per year:

$$\sum_{t=15}^{\infty} \frac{4000 * 1600}{(1+r)^t}$$

$$= \frac{1}{(1+r)^{14}} \sum_{t=1}^{\infty} \frac{4000 * 1600}{(1+r)^t}$$

$$= \frac{1}{(1+r)^{14}} * \frac{4000 * 1600}{r}$$

$$= \$27M$$

- Tax revenue from higher-income families:

$$\sum_{t=6}^{\infty} \frac{\$40M}{(1+r)^t}$$

$$= \frac{1}{(1+r)^5} \sum_{t=1}^{\infty} \frac{\$40M}{(1+r)^t}$$

$$= \frac{1}{(1+r)^5} * \frac{\$40M}{r}$$

$$= \$340M$$

Summing it up, the total economic benefit of the project is:

$$\$27M + \$340M = \$367M$$

Alternatively, you can argue that we want to include the benefit of the full increase in student incomes, not just their tax revenue, to be consistent with including student and teacher time costs (though you could also argue that those have fiscal repercussions on the government budget constraint). Then, the benefits are:

- Increase in student lifetime earnings: $80,000 per kid per year, and 1600 kids per year:

$$\begin{aligned} &\sum_{t=15}^{\infty} \frac{80000 * 1600}{(1+r)^t} \\ &= \frac{1}{(1+r)^{14}} \sum_{t=1}^{\infty} \frac{80000 * 1600}{(1+r)^t} \\ &= \frac{1}{(1+r)^{14}} * \frac{80000 * 1600}{r} \\ &= \$545M \end{aligned}$$

- Tax revenue from higher-income families:

$$\begin{aligned} &\sum_{t=6}^{\infty} \frac{\$40M}{(1+r)^t} \\ &= \frac{1}{(1+r)^{5}} \sum_{t=1}^{\infty} \frac{\$40M}{(1+r)^t} \\ &= \frac{1}{(1+r)^{5}} * \frac{\$40M}{r} \\ &= \$340M \end{aligned}$$

Summing up this alternative, the total economic benefit of the project is:

$$\$545M + \$340M = \$885M$$

Grading notes: 1 point for tax revenues from student lifetime earnings (or student lifetime earnings), 1 point for personnel costs, 1 point for tax revenues from higher-income families, 1 point for summing it all up consistently.

(b) (4 points) Now assume you work for the federal Department of Education and are deciding whether to provide a grant to the city of Cambridge, and they have proposed to use it to remodel their middle schools. Calculate each of the economic benefits associated with the project for *society as a whole*. Then compute the total benefit of the project to society.

Solution: Now, we only want to consider the effect on student lifetime earnings and not the increase in the tax base from higher-income families moving into Cambridge, since those families are leaving some other city's tax base. So, if we are only counting the increase in tax revenue, the economic benefit to society is only $27M compared to the cost of $336M. On the other hand, if we are assuming that the increase in student wages is due to a true increase in human capital, we want to count the full $545M.
Grading notes: 2 points for excluding higher tax base, 2 points for the right conclusion based on their assumptions.

(c) (2 points) Would the city of Cambridge want to embark on this project? Should the federal government give them a grant to do so? Why or why not?

Solution: Regardless of whether the city is using total societal benefits and costs, or just government budget benefits and costs, the city of Cambridge would like to do the project because their benefit ($367M or $885M, depending on your assumptions) is greater than the total cost ($336M or $322M, depending on your assumptions). (Note that technically, if you are not counting the increase in student incomes as a benefit, you should also not count the time costs, but we tried to give credit for any reasonable assumptions made).

If the federal government is basing their benefit calculation on the increase in tax revenue, they should not offer the grant, because the benefits are mostly accruing to Cambridge at the expense of other school districts, and the societal benefit ($27M) is much less than the total cost. But if the federal government is basing their benefit calculation on the increase in student incomes (assuming this is an increase in productivity, and not coming at the expense of others outside Cambridge), they should offer the grant, because the increase in productivity far outweighs the cost.
Grading notes: 1 point for each conclusion. If they made a mistake with the numbers and but draw the correct conclusion from their numbers, full credit.

4. (4 points) Now, imagine an ordinance passed and the city must pay all contractors at least $30 per hour. How does this change the cost-benefit analysis, and why?

Solution: This doesn't affect the cost-benefit analysis, because it only affects the accounting cost of the project. The cost-benefit analysis uses the equilibrium wage regardless of the wage paid because it is concerned with opportunity cost, and the equilibrium wage is still $25 per hour. The additional $5 is a transfer from the government to the construction workers.
Grading notes: 2 points for not changing the cost-benefit analysis, 2 points for mentioning the equilibrium wage being what matters.

5. (3 points) Imagine that the same exact project (with the same exact costs and benefits) was proposed in 4 middle schools in the Boston Public School system. Average household incomes of public school students in Boston are much lower than in Cambridge. Why might the federal DOE decide to give the grant to Boston Public Schools instead of Cambridge Public Schools?

Solution: BPS serves more students who are below the poverty line and/or receive food assistance; on average BPS students come from families that earn half the annual income of the average CPS student. The grant administrator may weight the benefits more highly for a district that serves a less advantaged population.

Grading notes: 3 points for any explanation related to distributional effects. 1 point for only discussing less uncertainty over the benefits in BPS; that's not really applicable here but is one reason you might prefer one project proposal to another.

6. (4 points) The DOE decided to provide a fixed $335M grant to Boston Public Schools on the condition that they use it to remodel their four oldest middle schools.

   (a) (2 points) What type of grant is the DOE providing to BPS?

   Solution: This is a conditional block grant: a fixed amount of money earmarked for a particular purpose.

   Grading notes: 2 points for conditional block grant. 1 point if they only mention a block grant.

   (b) (2 points) Why do you think the DOE is providing this grant instead of some other type of transfer?

   Solution: An unconditional block grant would not necessarily lead to increased spending in education, and certainly may not lead to the financing of this whole project. A matching grant changes the substitution patterns between paying for this project and other educational spending – and the DOE doesn't want to incentivize over-spending on this project relative to other educational investments.

   Grading notes: 1 point for why this is superior to a matching grant, 1 point for why this is superior to an unconditional block grant.