MIT 14.41 – Problem Set 3

Due October 21, 2022
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QUESTION 1: Education [38 points]

Consider a society with four types of families. Each family chooses s, an amount of quality schooling, and c, all other consumption. Assume that c is a numeraire good with price 1 and the price of one more unit of school quality is $p = \frac{1}{4}$. The four types of families are as follows:

$$y_a = 100; u_a = ln(c) + 4ln(s)$$

$$y_b = 100; u_b = ln(c) + ln(s)$$

$$y_c = 40; u_c = ln(c) + ln(s)$$

$$y_d = 40; u_d = ln(c) + 4ln(s)$$

There are 400 families of type A, 100 of type B, 200 of type C, and 300 of type D.

Assume that the future income of a child is $y^1 = \frac{y^0}{2} + \left(\frac{s}{4}\right)^{\frac{3}{2}}$, where y^0 is their household income when they are a child.

1. (2 points) Do parents' preferences take children's earnings fully into account when they choose their children's level of schooling? How can you tell?

> **Solution:** No, their utility functions do not include terms that accurately reflect the effect of schooling on their children's future income. Heterogeneity in how much parents care about s could come from how much they take this benefit into account, i.e. how "selfish" they are, or from a variety of other things.
> **Grading notes:** 1 point for "no," 1 point for the children's income term not showing up in parents' utility functions.

2. (4 points) First, let's solve for each family's decision when there is no public schooling. Find each type of family's chosen c, s.

> **Solution:** Maximization problems:
>
> - A: $\max_s ln(100 - \frac{s}{4}) + 4ln(s)$
> - B: $\max_s ln(100 - \frac{s}{4}) + ln(s)$
> - C: $\max_s ln(40 - \frac{s}{4}) + ln(s)$

- D: $\max_s ln(40 - \frac{s}{4}) + 4ln(s)$

FOCs and rearranging:

- A: FOC yields $\frac{-p}{100-ps} + \frac{4}{s} = 0 \rightarrow s = 320, c = 20$
- B: FOC yields $\frac{-p}{100-ps} + \frac{1}{s} = 0 \rightarrow s = 200, c = 50$
- C: FOC yields $\frac{-p}{40-ps} + \frac{1}{s} = 0 \rightarrow s = 80, c = 20$
- D: FOC yields $\frac{-p}{40-ps} + \frac{4}{s} = 0 \rightarrow s = 128, c = 8$

Grading notes: 1 point for each solution. Half credit for correct FOCs but math mistake.

3. Now, assume the government introduces public schools which provide the same quality schooling as choosing $s = E$, for free. If a family chooses to send their child to public school, they must consume exactly E education.

   (a) (2 points) Under what conditions on E will each type of family choose to send their child to public school?

   Solution:

   - A: $ln(100) + 4ln(E) > ln(20) + 4ln(320) \rightarrow E > 214$
   - B: $ln(100) + ln(E) > ln(50) + ln(200) \rightarrow E > 100$
   - C: $ln(40) + ln(E) > ln(20) + 4ln(80) \rightarrow E > 40$
   - D: $ln(40) + 4ln(E) > ln(8) + 4ln(128) \rightarrow E > 86$

   Grading notes: 0.5 points for each solution

   (b) Suppose the government set $E = 45$

   i. (2 points) Which families send their children to public school?

   Solution: Only family C

   Grading notes: 2 points for correct answer; full credit if answer is correct given their expressions in the previous part.

   ii. (2 points) What does this policy do to total productivity (measured by total future income of children)?

   Solution: Children's income from families A, B, and D are unaffected. Children from families of type C would have $y_1 = 20 + \left(\frac{45}{4}\right)^{\frac{3}{2}} = 57$ when there are low-quality public schools compared to $y_1 = 20 + \left(\frac{80}{4}\right)^{\frac{3}{2}} = 109$ when there are no public schools. So, total productivity decreases by $52 * 200 = 10,400$ as an outcome of the government policy to implement low-quality public schools.

Grading notes: 2 points for correct solution, 1 point if say that total productivity decreases but don't have mathematical solution.

iii. (2 points) What happens to inequality in this society? Here and for the rest of the problem, think about inequality in terms of the range of **children's future incomes**.

Solution:

- The children of family type A have income $y_1 = 50 + \left(\frac{320}{4}\right)^{\frac{3}{2}} = 766$ in either case
- The children of family type B have income $y_1 = 50 + \left(\frac{200}{4}\right)^{\frac{3}{2}} = 404$ in either case
- The children of family type D have income $y_1 = 20 + \left(\frac{128}{4}\right)^{\frac{3}{2}} = 201$ in either case

The range of incomes in society is increased by the introduction of low-quality public schools; those whose parents are at the bottom of the income distribution and care less about education have their relative incomes fall compared to what would have happened in the absence of public schools.

Grading notes: 2 points for the correct answer; 1 point if they just say it increases but don't give rationale or intuition.

(c) Now instead suppose the government set $E = 150$

i. (2 points) Which families send their children to public school?

Solution: Families B, C, D

Grading notes: 2 points for correct answer; full credit if answer is correct given their expressions in the previous part.

ii. (4 points) What does this policy do to total productivity (measured by total children's future income), relative to when there were no public schools?

Solution:

- Children's income from families A are unaffected.
- Children from families of type B would have $y_1 = 50 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 280$ when there are high-quality public schools compared to $y_1 = 50 + \left(\frac{200}{4}\right)^{\frac{3}{2}} = 404$ when there are no public schools.
- Children from families of type C would have $y_1 = 20 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 250$ when there are high-quality public schools compared to $y_1 = 20 + \left(\frac{80}{4}\right)^{\frac{3}{2}} = 109$ when there are no public schools.
- Children from families of type D would have $y_1 = 20 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 250$ when there are high-quality public schools compared to $y_1 = 20 + \left(\frac{128}{4}\right)^{\frac{3}{2}} = 201$ when there are no public schools.

So, total productivity increases by $(250 - 201) * 300 + (250 - 109) * 200 - (280 - 404) * 100 = 55,300$ as an outcome of the government policy to implement high-quality public schools.

Grading notes: 4 points for correct solution, 1 point if say that total productivity increases but don't have mathematical solution. 3 points if the steps are correct but there is a mathematical error. Full credit if answers are correct given previous solutions.

iii. (2 points) What happens to inequality in this society?

Solution: Based on the results in the previous part, high-quality public schools have increased children's future incomes for those with parents at the bottom of the distribution, and decreased children's future incomes for some of those with parents at the top of the distribution (though this hasn't affected the top end of the range, since children of type A still earn the most). Thus, there is less inequality than in the case with no public schools (or low-quality public schools!). The overall income range is smaller, and more children who grow up at the bottom of the distribution are moved up than the number who grow up at the top move down.

Grading notes: 3 points for the correct answer; 1 point if they just say it decreases but don't give rationale or intuition.

(d) (4 points) Discuss how and why parts (b) and (c) of this question differ in terms of productivity and inequality.

Solution: In part (b), the government implements low-quality public schools and only children from the poorer families whose parents don't care as much about education go to those schools. The provision of these schools actually crowds out schooling that these kids would have gotten absent the public schools, so total productivity goes down and inequality increases. However, when the government implements high-quality public schools and children from the top and bottom of the income distribution attend them, overall productivity increases and inequality decreases. There is still crowd-out (kids from families of type B get less schooling) but it is offset but large increases in the amount of schooling that kids from the bottom of the income distribution get.

Grading notes: 2 points for discussing crowd out (1 point for crowd out in (b), 1 point for how this compares to crowd-out in (c)), 2 points for intuition about who goes to public school in each case and relating this to the effect on productivity/inequality.

4. Instead of introducing public schools, suppose that in addition to their income, each family gets voucher of value V dollars from the government that they can use on education.

(a) (4 points) Write down each family's utility maximization problem, and solve for each family's choice of s and c (given V).

Solution:

- A: $\max_s ln(100 + V - ps) + 4ln(s)$ s.t. $ps \geq V$
- B: $\max_s ln(100 + V - ps) + ln(s)$ s.t. $ps \geq V$
- C: $\max_s ln(40 + V - ps) + ln(s)$ s.t. $ps \geq V$
- D: $\max_s ln(40 + V - ps) + 4ln(s)$ s.t. $ps \geq V$

Taking FOCs and solving:

- A: FOC yields $\frac{-p}{100+V-ps}+\frac{4}{s}=0 \rightarrow s^{\star}=\frac{4(400+4V)}{5}, c^{\star}=\frac{100+V}{5}$
- B: FOC yields $\frac{-p}{100+V-ps}+\frac{1}{s}=0 \rightarrow s^{\star}=\frac{4(100+V)}{2}, c^{\star}=\frac{100+V}{2}$
- C: FOC yields $\frac{-p}{40+V-ps}+\frac{1}{s}=0 \rightarrow s^{\star}=\frac{4(40+V)}{2}, c^{\star}=\frac{40+V}{2}$
- D: FOC yields $\frac{-p}{40+V-ps}+\frac{4}{s}=0 \rightarrow s^{\star}=\frac{4(160+4V)}{5}, c^{\star}=\frac{40+V}{5}$

Note: these are all conditional on $ps^{\star} \geq V$. If that condition for each family is not met, then they set $s = 4V$ and $c = y$ to max utility: i.e. if

- A: $V \geq 400 \rightarrow s^{\star} = 4V$ and $c^{\star} = 100$
- B: $V \geq 100 \rightarrow s^{\star} = 4V$ and $c^{\star} = 100$
- C: $V \geq 40 \rightarrow s^{\star} = 4V$ and $c^{\star} = 40$
- D: $V \geq 160 \rightarrow s^{\star} = 4V$ and $c^{\star} = 40$

Grading Notes: 0.5 points for each family's maximization problem and 0.5 points for each family's choice of s and c. Minus 1 point if they don't discuss the conditions under which they set $s = 4V$ (but they don't need to explicitly calculate in each case).

(b) (4 points) What level of schooling would each family choose if $V = 11.25$ *dollars*? How would this affect productivity and inequality, relative to the world without any public education or voucher system? (Note that, in terms of units of schooling, the voucher is worth $4 \times 11.25 = 45$ units)

Solution:

- A would choose $s = 356$, up from $s = 320$ in a world without vouchers or public schools, spending most of the voucher to purchase 36 more units of education quality and the rest to purchase (45-36)/4 = 2.25 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 890$.
- B would choose $s = 222.5$, up from $s = 200$ in a world without vouchers or public schools, spending half of the voucher to purchase 22.5 more units of quality education and the other half to purchase 5.625 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 465$.
- C would choose $s = 102.5$, up from $s = 80$ in a world without vouchers or public schools, spending half of the voucher to purchase 22.5 more units of quality education and the other half to purchase 5.625 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 150$.
- D would choose $s = 164$, up from $s = 128$ in a world without vouchers or public schools, spending most of the voucher to purchase 36 more units of education quality and the rest to purchase 2.25 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 283$.

Since all students are getting more education, productivity goes up unambiguously. It goes up by $(890 - 766) * 400 + (465 - 404) * 100 + (150 - 109) * 200 + (283 - 250) * 300 = 73,800$. Inequality has also increased, if we look at the range of the incomes in society, but mobility is greater (i.e. those at the bottom can get relatively closer to the top by investing more in education, compared to under public provision of education where they are more "stuck.")
Grading Notes: 2 points for correct calculations of the four levels of equilibrium schooling. 1 point for productivity going up unambiguously (or the calculation, don't need both). 1 point for saying that inequality has increased by the metric of the range of incomes in society OR that mobility is greater.

5. Now, let's think about the differences you found in the effects of the policies in part 2 and part 3.

   (a) (2 points) If the government is politically or financially constrained and can only offer $E = 45$ or $V = 11.25$ (worth 45 units of schooling), which should they do? Comment on the different effects of these policies on productivity and inequality.

   Solution: Public education and vouchers of the same size (45) have very different effects on productivity and inequality: this is because at this level, the government can only provide low-quality public schools that only the poorest children whose parents don't care as much about education will attend. And, because of the increase in consumption available, children from these families actually get less education than they would in the private market – public education crowds out private education. On the other hand, spending the same amount of money on vouchers leads all families to invest more in education, increasing productivity. Vouchers solve this crowd-out problem, holding the benefit offered to families fixed. Both increase inequality, though vouchers do so by more because all families get the benefit regardless of their income, and the government ends up paying some of the private school costs that families would have incurred in the absence of the program.
   Grading notes: 1 point for correct observations about levels of schooling and productivity, and 1 points for correct observations about inequality. 1 point if they draw the right conclusion but don't provide logic.

   (b) (2 points) If the government is unconstrained, should they offer the the policy you recommended in (a) or $E = 150$? Comment on the different effects of these policies on productivity and inequality.

   Solution: High-quality education can increase productivity (though not as much as lower-cost vouchers) and lower inequality, whereas vouchers increase inequality as described above. Which policy is preferred will depend on the government's social welfare function.
   Grading notes: 1 point for correct observations about levels of schooling and productivity, and 1 points for correct observations about inequality. 1 point if they draw the right conclusion but don't provide logic.

QUESTION 2: Selection in Health Insurance Markets [27 points]

Suppose there are three types of people in the world:

- Type A: People at high risk of having serious complications from Covid-19, who have a large probability $p_h \in (0, 1)$ of being hospitalized due to Covid in a given month, who have standard levels of risk aversion

- Type B: People at low risk of having serious complications, who have a smaller probability $p_l \in (0, p_h)$ of being hospitalized, who have standard levels of risk aversion
- Type C: People at low risk of having serious complications, who also have the smaller probability p_l of being hospitalized, who are more risk averse than the rest of the population.

Everyone has utility of the form:

$$u = \frac{c^{1-\eta}}{1-\eta}$$

The various levels of risk aversion are represented by types A and B having $\eta = 0.55$ and type C having $\eta = 0.75$

A fraction α of people are type A, a fraction β are type B, and the remainder are type C. Assume that everyone has an income stream of $2w$ and health care costs are w for anyone hospitalized with Covid.

1. (5 points) Initially suppose that competitive health insurers offer insurance contracts of the following form: If a person pays $m \geq 0$ in premiums, they receive a payout mb when they are hospitalized due to Covid. And, insurers can observe what type of person one is, so they can offer different contracts to each type. What contracts b_A, b_B, and b_C arise in equilibrium for each type of person, and what premiums m_A, m_B, and m_C are chosen by each type? (*Hint*: In a competitive equlibrium in an insurance market, insurers make zero profits).

Solution: Given competitive markets, the contracts will be actuarially-fair. For type A:

$$0 = (1 - p_h) * m + p_h * (m - mb) = 1 - p_h b \rightarrow b_A = \frac{1}{p_h}$$

For types B and C:

$$0 = (1 - p_l) * m + p_l * (m - mb) = 1 - p_l b \rightarrow b_B, b_c = \frac{1}{p_l}$$

Given these prices, each person of type $t \in \{A, B, C\}$ solves:

$$\max_{m \geq 0} p_t u(2w - w + m(\frac{1}{p_t} - 1)) + (1 - p_t) u(2w - m)$$

Because of actuarially-fair pricing, the solution will feature full insurance. So

$$2w - w + m(\frac{1}{p_t} - 1) = 2w - m \rightarrow m = wp_t$$

So, $m_A = wp_h, m_B = m_C = wp_l$

Grading notes: 2 points for correct contracts, 3 points for correct premiums (either by optimizing or arguing full insurance). 1 point if calculations for contracts are wrong but only depend on p's i.e. if $b_B = b_C$. Full credit for premiums if they are correct given their response to the contracts.

2. (2 points) In this market, what is the effect on premiums of a government policy that mandates full insurance (if a person pays some amount m in premiums charged by the insurance company, their hospital bills are paid completely by the insurer).

Solution: The previous part had full insurance contracts, so the policy has no effect and each person has the same willingness to pay as before. Insurers charge $m_A = wp_h, m_B = m_C = wp_l$
Grading notes: 2 points for no effect on premiums from previous part because the previous part had full insurance contracts.

3. (5 points) Next, we will study a setting where full insurance continues to be mandated, but insurers can only offer one type of contract to all people. First, what is the highest premium at which each type of person would buy insurance? Rank the types in terms of their demand for insurance. Under what condition will people of type C demand more insurance than those of type A?

Solution: A person of type t purchases insurance if and only if it yields higher expected utility than going uninsured:

$$u(2w-m) > p_t u(2w-w) + (1-p_t)u(2w)$$

$$\frac{(2w-m)^{1-\eta}}{1-\eta} > \frac{p_t w^{1-\eta}}{1-\eta} + \frac{(1-p_t)(2w)^{1-\eta}}{1-\eta} \rightarrow m < \bar{m}_t = w\left(2 - \left(p_t + 2^{1-\eta}(1-p_t)\right)^{\frac{1}{1-\eta}}\right)$$

Note that $\bar{m}_t$ is increasing in p_t, so holding η fixed, $\bar{m}_A > \bar{m}_B$ (higher-risk people are willing to pay a higher premium). And, $\bar{m}_t$ is increasing in η, so people who are more risk averse are always willing to pay a higher premium conditional on p_t ($\bar{m}_C > \bar{m}_B$). $\bar{m}_A > \bar{m}_C$ under the condition

$$p_h + 2^{0.25}(1-p_h) > \left(p_l + 2^{0.45}(1-p_l)\right)^{\frac{0.25}{0.45}}$$

Grading notes: 3 points for the solution for $\bar{m}_t$ (or correct answers for each type), 2 points for the correct ranking of A vs B and B vs C (1 for ranking, 1 for explanation.

4. When insurers offer a single premium m, three outcomes are possible: a *pooling equilibrium* where all three types of people purchase insurance, a *two-buyer separating equilibrium* in which two types purchase insurance, or a *one-buyer separating equilibrium* in which one type purchases insurance. In either case, equilibrium requires that insurance firms earn zero profits.

   (a) (3 points) Let's start with a pooling equilibrium. Solve for the premium at which an insurer would be willing to offer a contract in a pooling equilibrium. Under what condition(s) are all three types willing to pay this price?

Solution: In a pooling equilibrium, the zero-profit condition is:

$$\alpha\left[p_h(m-w) + (1-p_h)(m)\right] + (1-\alpha)\left[p_l(m-w) + (1-p_l)(m)\right] = 0$$

$$\alpha(m - p_h w) + (1-\alpha)(m - p_l w) = 0 \rightarrow m = \left[\alpha p_h + (1-\alpha)p_l\right] w$$

So, type A always purchases insurance because this is less than what they are willing to pay in part (1).

Type B and C purchase insurance when the equilibrium premium is lower than $\bar{m}_B$ and $\bar{m}_C$, respectively:

$$B: \left[\alpha p_h + (1-\alpha)p_l\right] w < w\left(2 - \left(p_t + 2^{0.45}(1-p_t)\right)^{\frac{1}{0.45}}\right)$$

$$C : [\alpha p_h + (1-\alpha)p_l]\, w < w\left(2 - \left(p_t + 2^{0.25}(1-p_t)\right)^{\frac{1}{0.25}}\right)$$

Grading Notes: 1 point for zero-profit condition, 1 point for type A always purchasing, 1 point for type B and C inequalities.

(b) (4 points) Now, let's consider a two-buyer separating equilibrium. Which two types will purchase insurance in this type of equilibrium? Solve for the premium at which an insurer would be willing to offer just those two types a contract. Under what condition(s) are only these two types willing to buy insurance?

Solution: The market features a separating equilibrium with two types purchasing insurance when type C is willing to buy, but type B is not (since $\bar{m}_C > \bar{m}_B$).

The new zero-profit condition is:

$$\alpha\left[p_h(m-w) + (1-p_h)(m)\right] + (1-\alpha-\beta)\left[p_l(m-w) + (1-p_l)(m)\right] = 0$$

$$\rightarrow m = \frac{w}{1-\beta}\left[(1-\alpha-\beta)p_l + \alpha p_h\right]$$

Note that this is always less than or equal to what type A is willing to pay in part (1):

$$\frac{w}{1-\beta}\left[(1-\alpha-\beta)p_l + \alpha p_h\right] \leq w p_h$$

Rearranges to $(1-\alpha-\beta)p_l \leq (1-\alpha-\beta)p_h$ which is always true.

Similar to above, C will buy insurance if

$$[\alpha p_h + (1-\alpha)p_l]\, w < w\left(2 - \left(p_t + 2^{0.25}(1-p_t)\right)^{\frac{1}{0.25}}\right)$$

And B will *not* buy insurance when

$$[\alpha p_h + (1-\alpha)p_l]\, w > w\left(2 - \left(p_t + 2^{0.45}(1-p_t)\right)^{\frac{1}{0.45}}\right)$$

Then there will be a separating equilibrium where only types A and C buy insurance.

Grading Notes: 1 point for having high-risk and highly risk-averse types buy insurance in this case, 1 point for the new zero-profit condition, 1 point showing that A will always be willing to buy and the inequality under which C also buys insurance, 1 point for the inequality describing the case where this is an equilibrium because B does not want to buy insurance.

(c) (3 points) Finally, let's consider a one-buyer separating equilibrium. Which type will purchase insurance in this type of equilibrium? Solve for the premium at which an insurer would be willing to offer only this type a contract. Under what condition(s) is only this one type willing to buy insurance?

Solution: The market features this type of equilibrium when only A is willing to buy insurance.

Now, A is the only one buying insurance. Since only high-risk people are buying insurance, the new zero-profit condition is

$$\alpha\left[p_h(m-w) + (1-p_h)(m)\right] = 0 \rightarrow m = w p_h$$

So, the separating equilibrium in which only type A's buy insurance has $m = w p_h$, the same as they would be offered if the firm could observe types.

Reversing the inequality for type C from above, type C will not want to buy if:

$$wp_h > [\alpha p_h + (1-\alpha)p_l]\, w < w\left(2 - \left(p_t + 2^{0.25}(1-p_t)\right)^{\frac{1}{0.25}}\right)$$

This also implies that the price will always be above B's willingness to pay, i.e.

$$wp_h > [\alpha p_h + (1-\alpha)p_l]\, w < w\left(2 - \left(p_t + 2^{0.45}(1-p_t)\right)^{\frac{1}{0.45}}\right)$$

Grading Notes: 1 point for only C buys insurance, 1 point for the new zero-profit condition and $m = wp_h$, 1 point for condition under which C and B do not buy insurance.

5. (3 points) Compare the equilibria in parts 3 and 4 (without type-specific contracts) to the equilibrium in part 2 (with type-specific contracts). Who is better-off, and who is worse-off, in parts 3 and 4 relative to part 2?

Solution: The low-risk people (type B and C) are worse off in any of the three equilibria because they either pay a higher price for full insurance than they do in (2) or they don't get any insurance at all. The high-risk, low risk-aversion people (type A) are better off in 4a and 4b, because in both cases they pay lower premia for full insurance than they otherwise would (they are subsidized by less risky people). They have the same outcomes in 4c as they do in part 2.

Grading notes: 1 point for each type.

6. (2 points) Based on this example, what can you say about how variation in risk aversion affects the feasibility of a private market for health insurance? What is the intuition behind this?

Solution: Low-risk, highly risk-averse individuals prolong the existence of and/or increase the feasibility of a functioning market for private health insurance. They are willing to subsidize the high-risk types at higher prices than the types who are similarly risky and less risk averse. These relatively healthy people place more value on consumption smoothing, so they get more value out of insurance. **Grading notes:** 1 point for prolonging market or increasing its feasibility; 1 point for intuition about subsidizing the high-risk types at higher prices.

QUESTION 3: Moral Hazard [35 points]

Ken is at risk of contracting a disease. With probability π, he will end up getting sick, and with probability $1-\pi$ he will stay healthy. If he contracts the disease, treating him will cost \$$c$. Assume that the financial cost is the only cost of the disease, so that he does not lose any additional utility from the disease. His income is $y > c$. Without insurance, his expected utility is therefore given by

$$E[u] = \pi u(y-c) + (1-\pi)u(y)$$

Now suppose that he has access to insurance. The insurance company will cover the full cost of Ken's healthcare if Ken gets sick. The company charges Ken a premium p for this insurance product, which he has to pay whether he gets sick or not. Thus if he buys insurance at price p, his expected utility is now

$$E[u] = \pi u(y-p) + (1-\pi)u(y-p) = u(y-p)$$

Throughout this question, the 'actuarially fair' price refers to the premium that is equal to the expected costs that the insurance company has to pay; if the insurance company chooses this premium, it will make zero profit in expectation. Also throughout this question, you can assume that if Ken gets exactly the same expected utility with and without insurance, he will choose to buy insurance.

1. (2 points) What is the actuarially fair premium p for this insurance product? Write down a condition in terms of c and π, and then calculate the value of the premium when $c = 500, \pi = 0.2$.

Solution:

Expected costs are πc, so the fair premium is $p = \pi c$; this implies $p = 100$ given the values for π and c.

2. (2 points) Write down a condition on p that must be satisfied for Ken to be indifferent between buying full insurance and not buying insurance, in terms of $u(\cdot), y, \pi, p$, and c.

Solution:

$$\pi u(y-c) + (1-\pi)u(y) = u(y-p)$$

Suppose that $y = 1000, c = 500, \pi = 0.2$. Suppose that Ken's utility function is

$$u(x) = \begin{cases} \ln(x) & \text{if } \sigma = 1 \\ \frac{1}{1-\sigma}x^{1-\sigma} & \text{if } \sigma \neq 1 \end{cases}$$

This means that σ is Ken's *coefficient of relative risk aversion*: higher σ means that he dislikes risk more. Note that $\ln(x)$ is the limit of $\frac{1}{1-\sigma}x^{1-\sigma}$ as $\sigma \to 1$. Throughout this question, when σ is not specified you should assume that $\sigma \geq 0$.

3. (6 points) What is the maximum amount that Ken will be willing to pay for full insurance:
   (a) When $\sigma = 0$?
   (b) When $\sigma = 1$?
   (c) When $\sigma = 2$?

Solution: Solving the condition in part 2 for p gives the general solution:

$$p = y - u^{-1}\left(\pi u(y-c) + (1-\pi)u(y)\right)$$

Substituting in the parameter values and different utility functions for different values of σ, we have:

(a): $p = y - (\pi(y-c) + (1-\pi)y) = \pi c = 100$

(b): $p = y - \exp\left(\pi \ln(y-c) + (1-\pi)\ln(y)\right) \approx 129.45$

(c): $p = y - \left(\pi(y-c)^{1-\sigma} + (1-\pi)y^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = y - \left(\pi(y-c)^{-1} + (1-\pi)y^{-1}\right)^{-1} \approx 166.67$

Now suppose that Ken can reduce his chances of contracting the disease by taking a new vaccine. The vaccine costs him \$$v$, and reduces his chances of getting the disease to $\frac{\pi}{2}$. It has no other direct effect on his utility (for instance, he does not get disutility from having to get an injection).

4. (2 points) First, imagine there is no insurance available. Write down the condition for the price v that must be satisfied for Ken to be indifferent between getting the vaccine and not getting the vaccine, in terms of $u(.), y, \pi, c$, and v.

Solution:

$$\frac{\pi}{2}u(y-v-c)+\left(1-\frac{\pi}{2}\right)u(y-v)=\pi u(y-c)+(1-\pi)u(y)$$

5. (2 points) Again suppose that $y = 1000, c = 500, \pi = 0.2$. What is the maximum amount that Ken would be willing to pay for the vaccine when $\sigma = 0$?

Solution: Need to solve the condition in part 4 for v. This can be done explicitly:

$$v=\frac{\pi}{2}c=50$$

6. (4 points) Suppose that the vaccine is actually available for \$20, and that insurance does not cover the cost of the vaccine, so that Ken has to pay \$20 for the vaccine even when he has insurance coverage. Why is there a moral hazard problem when Ken has full insurance? Does this moral hazard problem depend on Ken's value of σ? Note that $\sigma \geq 0$, and that when $\sigma > 0$, Ken's maximum willingness to pay for the vaccine when he doesn't have insurance will always be strictly greater than the amount you calculated in part 5.

Solution: With full insurance, Ken does not care whether he gets sick or not: his utility is the same either way, and he pays the same premiums either way. Thus he would not choose to pay for the vaccine with full insurance. However, at $v = \$20$ he would choose to pay for the vaccine if he had no insurance; thus the insurance causes a moral hazard response by causing him not to get the vaccine, raising his expected healthcare costs. This does not depend on σ, since Ken would buy the vaccine without insurance for any $\sigma \geq 0$

7. (6 points) Once again assume that $\pi = 0.2, c = 500, y = 1000$, and now assume $v = 20$. Taking into account the fact that Ken can choose to buy the vaccine when he does not have insurance, how much would he be willing to pay for full insurance when $\sigma = 1$? Comment on your answer and why it differs from your answer to part 3b.

Solution: Since $v = 20$ and we know that Ken is willing to pay at least \$50 for the vaccine from part 5, he will choose to buy the vaccine when he has no insurance. The indifference condition is thus

$$\frac{\pi}{2}u(y-c-v)+\left(1-\frac{\pi}{2}\right)u(y-v)=u(y-p)$$

Solving this for p, as before, gives

$$p = y - \exp\left(\frac{\pi}{2}\ln(y-c-v) + \left(1-\frac{\pi}{2}\right)\ln(y)\right) = 87.51$$

Note that this is now lower than the actuarially free premium, and lower than the amount from part 3b. This is because the ability to get the vaccine lowers the probability of being sick without insurance, but does not reduce premiums as moral hazard means Ken doesn't take the vaccine with insurance.

8. (3 points) Another insurance company offers a different insurance product; this product still covers the full cost of Ken's healthcare if he gets sick, but it also covers the cost of the vaccine. Assume that Ken will definitely (with 100% probability) get the vaccine when he doesn't have to pay extra for it. What would the actuarially fair premium for this insurance product be? Write an expression for p in terms of π, c, and v, and then calculate the value of p when $\pi = 0.2, c = 500, v = 20$.

Solution: The expected healthcare cost from getting sick is only $\frac{\pi}{2}c$, but now the insurance company also covers the healthcare cost, adding v to the cost; thus the fair premium is now

$$p = \frac{\pi}{2}c + v = 70$$

9. (4 points) Ken now has three options: an insurance product that covers the cost of the vaccine, an insurance product that does not cover the cost of the vaccine, and not buying insurance (but possibly buying the vaccine without insurance). Assume both insurance products charge the actuarially fair premiums. Rank these three options in order of how much utility Ken gets from each when $\sigma = 1$. (Hint: you do not need to calculate the level of utility; use your answers to parts 1, 7, and 8.)

Solution: We know from part 7 that Ken is willing to pay \$87.51 for insurance. Since the insurance company charges \$70 when it covers the cost of the vaccine from part 8, he gets \$17.51 of consumer surplus from it, so prefers this insurance product to not buying insurance. From part 1, the insurance company will charge \$100 when it does not cover the cost of the vaccine, meaning that Ken gets −\$12.49 of consumer surplus from that insurance product. Thus the ranking is

$$\text{Insurance with vaccine} \succ \text{No insurance} \succ \text{Insurance without vaccine}$$

10. (4 points) Suppose now that there is no vaccine for this disease, but Ken is instead able to reduce his risk of getting the disease by eating a restricted diet. Assume he would dislike this restricted diet compared to his ideal diet. Why might it be harder for the insurance market to address this form of moral hazard? (No math required for this part.)

Solution: In this case the activity is something that Ken actively dislikes, rather than something that he is indifferent about and dislikes paying for, so it wouldn't be sufficient for the insurance company to cover the costs of that diet. Furthermore, it is harder to monitor diet than whether or not Ken has got a vaccine. Both these factors mean that insurance companies probably wouldn't be able to incentivise Ken to eat the restricted diet and reduce his health risk, meaning that there would be moral hazard and he would eat a less restricted diet when he has insurance.