MIT 14.41 – Problem Set 4

Due November 4, 2022
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QUESTION 1: Social Security [36 points]

Tegan makes decisions about two periods in her life: In her working period, she earns a wage w and can save an amount s at interest rate r (with $s \geq 0$). In her retirement period, she doesn't earn anything but consumes her savings (plus interest), and she cares some fraction $\delta \in [0, 1]$ about her retirement period consumption relative to her working period consumption. She knows that she won't have children and doesn't want to leave any money behind. So, she chooses c_w, an amount to consume during her working period, and c_r, an amount to consume during her retirement period, to maximize her lifetime utility:

$$U = ln(c_w) + \delta ln(c_r)$$

subject to the constraints that:

$$w = c_w + s$$

$$c_r = s(1 + r)$$

1. (5 points) How much does Tegan optimally save? What is Tegan's optimal consumption in each period?

Solution: Tegan solves

$$\max_s ln(w - s) + \delta ln(s(1 + r))$$

FOC wrt s:

$$\frac{-1}{w - s} + \frac{\delta(1 + r)}{s(1 + r)} = 0$$

Rearranges to:

$$s = \frac{\delta w}{1 + \delta}$$

Plugging into the expressions for c_w and c_r, $c_w = \frac{w}{1+\delta}$ and $c_r = \frac{\delta w(1+r)}{1+\delta}$

Grading notes: 1 point for correct maximization problem, 1 point for FOC, 1 point for correct c_w, 1 point for correct c_r, 1 point for correct c_s

2. (3 points) Why might the government be concerned that Tegan isn't saving enough?

Solution: The rationale that is consistent with this setup is that the government is paternalistic: they don't think Tegan will save enough for retirement. For example, they may think that she is myopic and her δ is too low. Note that while market failures in the annuity market is a rationale for government intervention in

the real world, the problem as it is described here does not feature any market failures.

Grading notes: 3 points for "paternalism" plus intuition behind it or for saying the government might think Tegan is using too small a δ. 1 point if just say "paternalism." Minus 0.5 points for discussing market failures in annuities, since that is not a rationale for this specific case.

3. For the reason(s) you described above, the government decides to help Tegan save for retirement. They require her to put some amount b in a retirement account during her working period, and they return back $b(1+r)$ during her retirement period. She can still choose to put some amount s into a personal savings account, which still returns $s(1+r)$ in the retirement period.

   (a) (2 points) Write down the problem that Tegan will solve now

   Solution: Tegan solves:

   $$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$
   $$\text{s.t. } w = c_w + s + b$$
   $$c_r = (s+b)(1+r)$$

   Grading notes: 2 point for correct set-up

   (b) (5 points) How does this government policy affect Tegan's chosen consumption and **private** savings levels, relative to the case without government intervention? What about her **total** savings? Why?

   Solution: The problem above can also be written as:

   $$\max_{s} ln(w - s - b) + \delta ln((s+b)(1+r))$$

   FOC wrt s:

   $$\frac{-1}{w-s-b} + \frac{\delta(1+r)}{(s+b)(1+r)} = 0$$

   Rearranges to:

   $$s = \frac{\delta w}{1+\delta} - b$$

   Plugging into the expressions for c_w and c_r, $c_w = \frac{w}{1+\delta}$ and $c_r = \left(\frac{(\delta w)}{1+\delta}\right)(1+r)$.
   Tegan puts less into her private savings account (a dollar of b reduces her personal savings by a dollar, until she sets $s = 0$). When $\frac{\delta w}{1+\delta} > b$, her outcomes are unchanged. When $\frac{\delta w}{1+\delta} < b$, the policy forces her to save more and consume less in the working period and consume more in the retirement period (i.e. smooths consumption).

   Grading notes: 1 point for correct s, 1 point for correct c_w and c_r, 1 point for government policy having no effect unless she is saving less than b to begin with, 2 points for in which case it increases saving, and smooths consumption.

   (c) (2 points) What is this type of social security called, and why is it a good idea for countries just setting up a social security system?

Solution: This is funded social security. It's primary benefit is that it pays retirees using the money that they contributed to the system, so it is always solvent (unless the money was invested badly).

Grading notes: 1 point for funded social security, 1 point for solvency

4. Now, assume that there are 2 otherwise identical people alive at any time, except one is in their working period and the other is in their retirement period. Instead of forcing individuals to save for the own retirement, the government has instituted the following: in their working years, an individual faces a social security tax at rate x, the proceeds of which is paid out to the individual currently in their retirement years. When that person retires, they receive the social security tax proceeds collected from the subsequent generation. Assume that wages are growing at the same rate as savings, so the workers who work when Tegan is retired are earning $w(1+r)$, and Tegan knows this will be true when she makes her initial consumption and savings decisions.

   (a) (2 points) Write down the problem that Tegan will solve now

Solution: Tegan solves:

$$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$

$$\text{s.t. } w - wx = c_w + s$$

$$c_r = s(1+r) + xw(1+r)$$

Grading notes: 2 point for correct set-up

   (b) (4 points) Is this policy economically different from the previous part? Why? What is the effect of this policy on Tegan's savings and consumption rates?

Solution: No – we've just replaced b with xw. Therefore, the policy has the same effect as above.

Grading notes: 1 point for no, 1 point for replacing b with xw, 2 point1 for policy having the same effect as before.

   (c) (5 points) Now imagine that wage growth was slower than the rate of returns on financial investments (though both are strictly greater than 0). In particular, wages are growing at a rate $\tilde{r} < r$. How would this affect Tegan's saving levels and consumption levels, relative to part (b)?

Solution: With the rate of wage growth $\tilde{r} < r$, Tegan solves:

$$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$

$$\text{s.t. } w - wx = c_w + s$$

$$c_r = s(1+r) + xw(1+\tilde{r})$$

Or equivalently:

$$\max_{s} ln(w - wx - s) + \delta ln(s(1+r) + wx(1+\tilde{r}))$$

FOCs:

$$\frac{-1}{w - wx - s} + \frac{\delta(1+r)}{s(1+r) + wx(1+\tilde{r})} = 0$$

Rearranging,

$$s = \frac{\delta w}{1+\delta} - \left(\frac{\delta(1+r)+1+\tilde{r}}{(1+\delta)(1+r)}\right) wx$$

The coefficient on xw is strictly less than 1, since it can be written as $\frac{\delta}{1+\delta} + \frac{1+\tilde{r}}{(1+\delta)(1+r)}$, which converges to 1 as $\tilde{r} \to r$, but with $\tilde{r} < r$ there is a smaller numerator in the right side fraction, so the total is less than one. So now a dollar of benefits crowds out less than a dollar of private savings.
Tegan consumes less in period 1 and period 2 than before:

$$c_w = \frac{w}{1+\delta} - wx\left(1 - \frac{\delta(1+r)+1+\tilde{r}}{(1+\delta)(1+r)}\right)$$

which is strictly less than $\frac{w}{1+\delta}$ since as we showed above, $\frac{\delta(1+r)+1+\tilde{r}}{(1+\delta)(1+r)} < 1$, and similarly:

$$c_r = \frac{\delta w(1+r)}{1+\delta} + wx\left(1 + \tilde{r} - \frac{\delta(1+r)+1+\tilde{r}}{1+\delta}\right)$$

which is strictly less than $\frac{\delta w(1+r)}{1+\delta}$ since the second term is always negative since $1 + \tilde{r} < 1 + r$.

Grading notes: 2 points for correct s. 2 points for now having less than full crowd-out. 1 point for lower consumption in both periods.

Now let's consider another decision affected by social security – the decision to retire. Sara lives in the Netherlands, and is deciding when to retire. Each year, she can earn a wage w if she works, of which she pays t in social security taxes, or, if she retires, she can receive αw in social security benefits for each year going forward. In year 0, she chooses whether or not to retire by maximizing her remaining lifetime income for the next N years:

$$U = \sum_{t=0}^{N} \delta^t ln(c_t)$$

5. (1 point) What is α called?

Solution: α is the replacement rate
Grading notes: 1 point for replacement rate

6. Write down an expression for Sara's lifetime utility if she retires in...
   (a) (1 point) ... year 0?

Solution:

$$U = \sum_{t=0}^{N} \delta^t ln(\alpha w)$$

Grading notes: 1 point for correct expression

(b) (1 point) ... year 1?

Solution:

$$U = ln(w(1-t)) + \sum_{t=1}^{N} \delta^t ln(\alpha w)$$

Grading notes: 1 point for correct expression

(c) (1 point) ... year 2?

Solution:

$$U = ln(w(1-t)) + \delta ln(w(1-t)) + \sum_{t=2}^{N} \delta^t ln(\alpha w)$$

Grading notes: 1 point for correct expression

7. (4 points) Sara is deciding whether to retire in year 0. What inequality should she solve to make this decision? Solve this inequality for an expression in terms of α,w, t, and/or δ, and explain the intuition behind the condition you get.

Solution: Sara retires in year 0 when her lifetime utility from doing so is greater than her lifetime utility from waiting a year.

$$\sum_{t=0}^{N} \delta^t ln(\alpha w) > ln(w(1-t)) + \sum_{t=1}^{N} \delta^t ln(\alpha w)$$

$$\delta^0 ln(\alpha w) > ln(w(1-t))$$

$$\alpha w > w(1-t)$$

$$\alpha > 1 - t$$

A high payroll tax or a high replacement rate both make it more likely that Sara retires in year 0. Sara will retire when she could earn more in benefits than she could working.
Grading notes: 1 point for set-up, 1 point for correct condition, 2 points for intuition.

QUESTION 2: Unemployment Insurance [36 points]

This question will ask you to think about issues in the design of unemployment insurance. To do this, we'll assume a model with two time periods and people who can be employed or unemployed. Employed people earn a wage of $w > 0$, and unemployed people receive an unemployment insurance payment of $b \geq 0$ from the government.

In the first period, a fraction $e_1 \in [0, 1]$ of people are employed and $1 - e_1$ are unemployed. In the second period, employed workers lose their jobs with probability π, which is fixed. Unemployed workers find a new job with probability

$$p = \bar{p} + \gamma x$$

where $\bar{p} \in [0, 1]$ and γ are constants. The workers can choose $x \in \left[0, \frac{1-\bar{p}}{\gamma}\right]$ by choosing how much effort they put into finding a job, but to do this they must pay a utility cost of x^2.

Workers have access to a consumption good c with price 1. Assume they cannot borrow or save, so in each period they spend all their income for that period – either their wage w or their benefit b – on c (this means we are assuming there is no self-insurance). Their utility from consuming c in a period is $u(c)$, where u is an increasing and concave function: $u'(c) > 0, u''(c) < 0$ for all c. Assume that workers do not discount the future. Then they care about their utility today as well as their expected utility in the next period. They will make choices to maximise the following function:

$$EU_u = u(b) + (\bar{p} + \gamma x)\, u(w) + (1 - (\bar{p} + \gamma x))\, u(b) - x^2$$

Throughout this question, you should assume that workers want to choose an interior solution for x (that is, their optimal choice of x satisfies $0 < x < \frac{1-\bar{p}}{\gamma}$)

1. (3 points) Derive an expression for an unemployed worker's optimal choice of x if the worker chooses an interior solution.

   Solution: Differentiating the expected utility function with respect to x gives the FOC

   $$\gamma\,(u(w) - u(b)) - 2x = 0 \iff x = \frac{\gamma}{2}\,(u(w) - u(b))$$

   Grading notes: 1 point for differentiating the expected utility function, 1 point for correct FOC, 1 point for correctly rearranging to get the expression for x.

2. (a) (2 points) What level of b would provide full insurance against unemployment for workers? What would effort be if the government set b at this level?

   Solution: Full insurance requires $b = w$ so that consumption is the same in insured and uninsured states. When this is the case, $u(w) = u(b)$ so the FOC from part 1 implies $x = 0$.
   Grading notes: 1 point for stating that $b = w$, 1 point for inferring that effort would be 0.

   (b) (4 points) Differentiate $p = \bar{p} + \gamma x$ with respect to b (your answer can be in terms of u', the derivative of u), and interpret in words what this means for how increases in benefits change the probability of unemployed people finding a job. How does this effect depend on γ?

   Solution:

   $$\frac{dp}{db} = \gamma \frac{dx}{db} = -\frac{\gamma^2}{2} u'(b)$$

   In words, this means that higher benefits reduce the probability of unemployed people finding a job (since $u'(b) > 0$), and that higher γ raises the effect of benefits on the probability of finding a job.
   Grading notes: 1 point for an answer of the form $\frac{dp}{db} = \gamma \frac{dx}{db}$, 1 point for correct value of $\frac{dx}{db}$, 1 point for saying that higher benefits lower the probability of unemployed people finding jobs, 1 point for saying that this effect is stronger when γ is higher.

   (c) (2 points) What do we call the effect you demonstrated in part (b)?

   Solution: This is an example of moral hazard: providing more insurance makes the bad state (unemployment) relatively more desirable, and so makes people take behaviour that increases their risk of being in the bad state (searching for jobs less).

Grading notes: 1 point for referring to moral hazard, 1 point for reasonable explanation of what the moral hazard is in this context.

3. (a) (3 points) What fraction of people will be employed in period 2, in terms of $\bar{p}, \pi, x, \gamma$, and e_1? Label this value e_2.

Solution: People are employed in period 2 if they were employed in period 1 and stayed employed, or if they were unemployed in period 2 and found a job. Thus

$$e_2 = e_1(1-\pi) + (1-e_1)p = e_1(1-\pi) + (1-e_1)(\bar{p} + \gamma x)$$

Grading notes: 1 point for answer that correctly identifies the number of employed people who keep their job is $e_1(1-\pi)$, 1 point for answer that correctly identifies the number of unemployed people who get a job is $(1-e_1)p$, 1 point for substituting in definition of p correctly and adding these quantities up.

(b) (3 points) Differentiate e_2 with respect to b. In words, what does this imply about how unemployment benefits affect the employment rate in period 2?

Solution:

$$\frac{de_2}{db} = -(1-e_1)\frac{\gamma^2}{2}u'(b)$$

So higher unemployment benefits lower the employment rate in period 2, as we would expect.

Grading notes: 2 points for correct derivative (given answer to part 2b), 1 point for correct interpretation that higher benefits lower the employment rate.

Now suppose that the government has the following loss function:

$$L = \alpha(1-e_2)^2 + (1-\alpha)(u(w) - u(b))^2$$

This means that the government cares about both getting the employment rate closer to 1 and about providing more insurance for workers by providing benefits closer to w. α is a parameter that measures how much the government cares about high employment relative to full insurance. The government's objective is to *minimise* this function.

4. (a) (2 points) What benefit level should the government choose when $\alpha = 0$, so that it only cares about providing full insurance to workers?

Solution: When $\alpha = 0$, the government should just choose $b = w$.

Grading notes: 1 point for identifying that full insurance is optimal but choosing some $b \neq w$.

(b) (2 points) What benefit level should the government choose when $\alpha = 1$, so that it only cares about getting the employment rate in period 2 as close to 1 as possible?

Solution: Since the employment rate is strictly decreasing in b, but $b \geq 0$, to maximise the employment rate the government will set $b = 0$.

Grading notes: 1 point for observing that employment will be higher when b is lower, 1 point for concluding that $b = 0$.

(c) (3 points) Intuitively (no math required), will increasing α (in the range between 0 and 1) increase or decrease the optimal level of b, and why?

> **Solution:** Increasing α raises how much the government cares about employment relative to insurance; since higher b lowers employment, higher α implies that the government will choose lower b.
>
> **Grading notes:** 1 point for correct sign, 1 point for reasonable explanation of intuition that higher b is bad for employment but good for insurance, 1 point for inferring that greater preference for employment implies lower b.

5. (a) (3 points) What benefit level should the government choose when $\gamma = 0$ (if $0 < \alpha < 1$), and why?

> **Solution:** When $\gamma = 0$ there is no moral hazard, since the unemployed cannot raise their probability of finding a job by searching more. Thus only the insurance motive matters, even though $\alpha > 0$, and the government will set $b = w$.
>
> **Grading notes:** 1 point for identifying no moral hazard, 1 point for concluding that only insurance motive matters even when $\alpha > 0$, 1 point for concluding that $b = w$.

(b) (3 points) Intuitively (no math required), when $0 < \alpha < 1$, will increasing γ increase or decrease the optimal level of b, and why?

> **Solution:** Higher γ will reduce the optimal level of b. Intuitively, higher γ means that raising b has a larger moral hazard effect on unemployment, while it has the same effect on the degree of insurance provided, so it becomes relatively less desirable and lower b will be optimal.
>
> **Grading notes:** 1 point for correct sign, 1 point for intuition that higher γ means more moral hazard but same insurance, 1 point for conclusion that optimal level of b will fall.

6. (6 points) During the peak of the COVID-19 pandemic, the federal government substantially increased unemployment insurance payments as part of the Coronavirus Aid, Relief and Economic Security (CARES) Act, in many cases paying people as much as they earned in their previous jobs (so that $w = b$). Before the pandemic, unemployment insurance payments were much lower so that usually $w > b$ and $w - b$ was reasonably large. Based on your answers to the previous parts of this question, suggest and explain *two* possible reasons why the US government might have temporarily increased the size of its unemployment insurance payments during the pandemic.

> **Solution:** One reason is that the government's objective may have changed: during the pandemic the government wanted to keep some businesses closed, and so temporarily cared less about having a lot of people in work relative to providing insurance. This corresponds to a decline in α, which raises the optimal level of b based on 2.4. A second reason is that the moral hazard effects of UI may have been lower in the pandemic; firms in many sectors may not have been hiring at all, meaning that workers could not get hired regardless of the amount of effort that they put into search. This corresponds to a decline in γ, which raises the optimal level of UI based on 2.5.
>
> **Grading notes:** up to 3 points for each reason: in both cases, 1 point for identifying the relevant parameter change and up to 2 points for a reasonable explanation of why the pandemic caused this parameter to change.

QUESTION 3: Social Insurance Potpourri (T/F/U) [28 points]

State whether each of the following claims is true, false, or uncertain and explain why in 2-5 sentences. No credit will be awarded without an explanation.

1. (7 points) **Social Security.** Social security helps individuals smooth consumption over time.

   Solution: True. Social security will only help consumption-smooth when it does not crowd out private savings. Indeed, evidence suggests that there is moderate ($0.30-0.40 reduction in private savings for each dollar of social security), but not full crowd-out, and that a substantial minority of individuals are not financially prepared for retirement. This means that social security does help smooth consumption.
   Grading notes: 1 point for true, 3 points for consumption smoothing if not full crowd out, 3 points for empirical evidence.

2. (7 points) **Unemployment Insurance.** One proposed reform to the unemployment insurance system is *worker self-insurance*, where workers contribute their own money to a savings account that is reserved for unemployment insurance, and can draw on this account if they lose their job.

   Claim 1: a worker self-insurance system would reduce the length of time that unemployed people spend looking for jobs on average.

   Claim 2: a worker self-insurance system would be more economically efficient than the current system.

   Solution:

   Claim 1: true. When workers have to self-insure, they will pay the full marginal cost of additional time in unemployment themselves. This makes it less desirable to spend time in unemployment than when unemployment is covered by government unemployment insurance, and thus will encourage people to take jobs quicker.

   Claim 2: uncertain/true. Worker self-insurance could increase efficiency by reducing moral hazard and time spent in unproductive leisure, based on the argument for claim 1. On the other hand, it could decrease efficiency if having to pay the cost of insurance reduces job match quality by encouraging people to take worse jobs quicker. However, evidence discussed in the textbook suggests that this theoretical argument is not empirically relevant – there is no clear evidence that higher UI raises job match quality – so the pro-efficiency effect of worker self-insurance may dominate.

   Grading notes:

   Claim 1: 1 point for 'true', 1 point for identifying that unemployment becomes more costly to workers, 1 point for arguing that this means people should take jobs quicker.

   Claim 2: 1 point for arguing that reduced moral hazard could increase efficiency, 1 point for arguing that reduced match quality could reduce efficiency. Can either get 2 points for saying 'true' and arguing that the empirical evidence suggests benefit generosity has little effect on match quality, or for saying 'uncertain' and arguing that the empirical question of which force matters more is not settled.

3. (7 points) **Disability Insurance.**

Research discussed in lecture and section 14.3 of the textbook, such as French and Song (2014), suggests that there is some evidence of moral hazard responses to disability insurance: some people who receive disability insurance are capable of working in jobs that suit their skills.

Claim: this evidence proves that the current system of assessing people for disability insurance is not strict enough, and that evaluators should require stronger evidence of disability before declaring someone eligible for DI.

Solution:

False. Making the evaluations stricter could reduce moral hazard responses to DI: if strong evidence is needed to claim DI, then only people who are truly disabled will be able to claim DI. This would increase efficiency by making sure that the government is not subsidising leisure and paying for people to stay out of the labor force when they could work, and would be desirable.

However, stricter evaluations could also potentially mean that people who are truly disabled become less likely to receive DI. This would be inefficient, and socially undesirable, because it would reduce the degree to which DI smooths consumption between disabled and non-disabled states of the world and only providing partial insurance for disability. From an equity perspective people who are truly disabled (so unable to work) are disproportionately poor, and reducing their ability to receive DI thus redistributes away from a poor group. So the evidence of moral hazard in DI is certainly not sufficient to claim that DI evaluations should be stricter.

Grading notes: 1 point for 'false'. 1 point for arguing that stricter evaluations reduce moral hazard, 1 point for arguing that this would increase efficiency, 1 point for arguing that stricter evaluations may prevent people who should get DI from getting it, 1 point for an explanation why this is undesirable in terms of efficiency (less insurance), 1 point for an explanation discussing equity, 1 point for arguing that this means the evidence is not sufficient to justify the claim.

4. (7 points) **Workers Compensation.** Two components of the current design of workers' compensation programs likely lead to more efficient outcomes than the status quo before states mandated that employers buy insurance against on-the-job accidents: (1) the fact that these are *no-fault* insurance policies and (2) the use of partial experience rating in determining firms' insurance premiums. Please respond T/F/U separately for each component (1) and (2).

Solution: (1) Uncertain. Before WC, workers had to sue firms when they experienced on-the-job injuries, and the costs that they paid to lawyers, etc. were a deadweight loss to society because the outcome only involved a transfer between two parties. Now, workers get paid their benefits without having to go to court to prove that their employer was at fault, a more efficient outcome if the same workers would be paid the same benefits in both cases. However, making it easier to get payments can also increase moral hazard, which could lead to less efficient outcomes.

(2) False. Before WC, (and if workers had sufficient bargaining power/legal resources) firms chose the efficient level of workplace safety to equate the marginal cost of making the workplace safer with the marginal benefit of reducing the costs of workplace injuries. With WC's partial experience rating, firms only pay a fraction of the cost of their employees' workplace injuries, distorting their choice of the level of workplace safety.

Grading notes: 0.5 points for (1) is uncertain, 1.5 points for reducing deadweight loss by reducing legal fees, 1.5 points for more moral hazard; 0.5 points for (2) is false, 1.5 points for firms equating marginal costs and benefits before WC, 1.5 points for how partial experience rating changes that equation.

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