MIT 14.41 – Problem Set 5

Due November 18, 2022
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QUESTION 1: Health economics [55 points]

Assume that each person has a probability of being hospitalized in the next year, η, and being hospitalized has medical costs of $L = 300$. For simplicity, assume that the risk types are uniformly distributed in the interval (0,1), with a total mass of 1. This technical assumption implies the following:

- Individual risk types lie in the interval [0,1]
- The fraction of people with risk type $\eta \leq x$ is given by x, with $x \in [0, 1]$
- The probability that an individual with risk type η is hospitalized is $Pr = \eta$

For example, 25% of people have less than a 25% chance of being hospitalized in the next year, 50% have less than a 50% chance of being hospitalized, etc. This is just the definition of a uniform distribution.

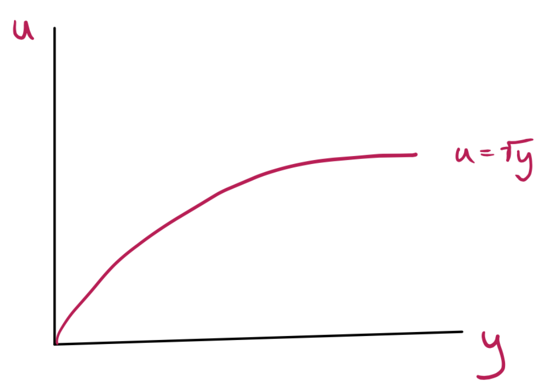
Each individual has a fixed income $W = 400$ and gets utility from consumption y (the price of consumption is 1), given by

$$U = \sqrt{y}$$

Finally, we will assume that an individual's risk type can be determined by a medical exam, so individuals know their own risk type.

1. (a) (1 point) Plot individual utility on a graph with utility on the vertical axis and consumption y on the horizontal axis

Solution:



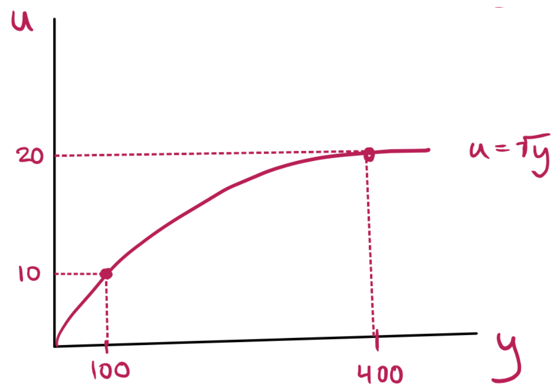
Grading notes: 1 point for correct shape

(b) (5 points) What is the level of consumption and resulting utility in the case that someone is hospitalized, and the case when they are not? Label these on your plot of utility against consumption.

Solution:

$$u(y|hospital) = \sqrt{W-L} - \sqrt{400-300} = 10$$

$$u(y|nohospital) = \sqrt{W} - \sqrt{400} = 20$$

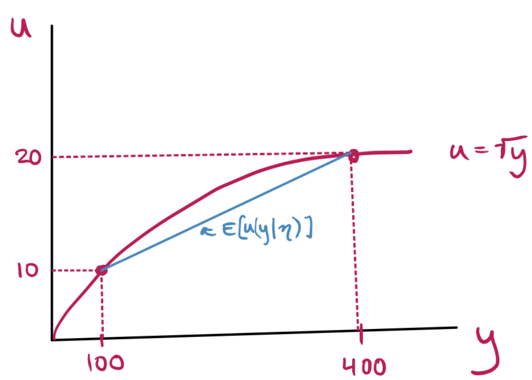


Grading notes: 1.5 points for each utility calculation, 1 point for correctly labeling each on graph

(c) (3 points) What is the expected utility of an individual with risk type η? Plot this on your graph.

Solution:

$$E[u(y|\eta)] = \eta\sqrt{W-L} + (1-\eta)\sqrt{W} = 10\eta + 20(1-\eta) = 20 - 10\eta$$



Grading notes: 1 point for correct set-up, 1 point for correct expression, 1 point for drawing the right line on the graph.

2. Suppose there is an insurance company that offers a policy that fully covers the cost of hospitalization. Assume the company can obtain each person's medical exam (i.e. the company knows each person's risk type).

(a) (2 points) What is the actuarially fair premium π_η for this insurance policy, for an individual of type η?

Solution: The actuarially fair price is equal to the expected cost of covering an individual. For an individual of risk type η, this is 300η.
Grading notes: 2 points for correct answer

(b) (4 points) Calculate the utility each individual would get from purchasing insurance at the actuarially fair price. Which individuals will choose to purchase insurance?

Solution:
$$u(y) = \eta\sqrt{W - \pi_\eta - L + L} + (1-\eta)\sqrt{W - \pi_\eta} = \sqrt{W - \pi_\eta} = \sqrt{400 - 300\eta}$$
A type η will buy insurance iff

$$\sqrt{400 - 300\eta} > 20 - 10\eta \rightarrow 400 - 300\eta > 400 - 400\eta + 100\eta^2 \rightarrow 0 > 100\eta(\eta - 1)$$

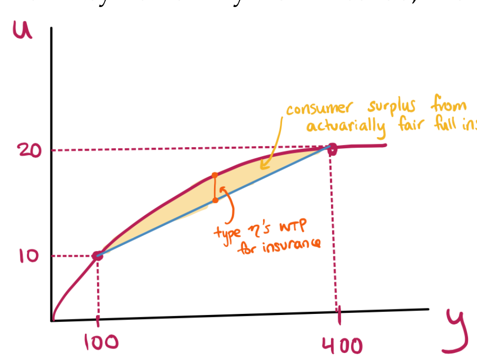
Which is always true for types $\eta < 1$, and type $\eta = 1$ is indifferent between insurance and no insurance.
Grading notes: 1 point for the utility from purchasing insurance at the actuarially fair price, 2 points for the right starting inequality, 1 point for any argument that all types will buy insurance.

(c) (1 point) How much does it cost the insurance company, on average, to cover an individual of type η?

Solution: It costs 300η to cover someone of risk type η
Grading notes: 1 for right answer

(d) (3 points) Is it socially efficient for an individual of type η to have insurance in this case? Why or why not? Who is it socially efficient to cover? Show this visually on your graph.

Solution: It is socially efficient to cover anyone (of any risk type) because an individual has a higher utility when they buy full insurance at an price of the expected cost of covering their hospital bills than when they do not buy insurance. So, it is socially efficient for everyone to have insurance.



Grading notes: 1 point for rationale that there is a higher benefit of buying insurance than the marginal cost of providing insurance, 1 point for saying is is efficient for everyone to have insurance, 1 point for showing either the consumer surplus to a person of type η or the total consumer surplus.

Now, suppose the insurance company can no longer obtain medical exams and it can't distinguish people's types. We want to know how much an insurer will charge for a full insurance policy, and how many people will buy the policy at that price. We also want to compare this to the socially efficient outcome. To do this, we need three things, which we will collect in the following questions:

- Consumer demand for insurance
- Marginal cost of providing insurance to a fraction Q of individuals
- The average cost of providing insurance to a fraction Q of individuals

3. We begin by deriving consumer demand for insurance, as follows:

(a) (4 points) First, what is the maximum price (aka premium) c_η that a person of type η is willing to pay for full insurance? (Hint: c_η will be a quadratic expression in terms of η).

> **Solution:** The maximum price a person is willing to pay is the price that makes them indifferent between purchasing insurance and not:
>
> $$E[u(y)|\eta] = U(W - c_\eta) \rightarrow 20 - 10\eta = \sqrt{400 - c_\eta} \rightarrow c_\eta = 400\eta - 100\eta^2$$
>
> **Grading notes:** 2 points for right indifference condition, 2 points for the right c_η.

(b) Using willingness to pay, derive the consumer demand curve using the following:

i. (1 point) At a price of p, who will buy insurance? Write an expression in terms of c_η and p. Label the "marginal person" (the "last" person to buy insurance at price p) to have risk equal to $\bar{\eta}$. In terms of $\bar{\eta}$, what is the price that the marginal person is willing to pay?

> **Solution:** At a price of p, everyone who has $c_\eta \geq p$ will buy. So the marginal person will have $c_{\bar{\eta}} = p$, or $p = 400\bar{\eta} - 100\bar{\eta}^2$.
> **Grading notes:** 0.5 points for $c_\eta \geq p$, 0.5 points for $p(\eta)$

ii. (1 point) What is Q, the fraction of people who buy insurance, in terms of $\bar{\eta}$?

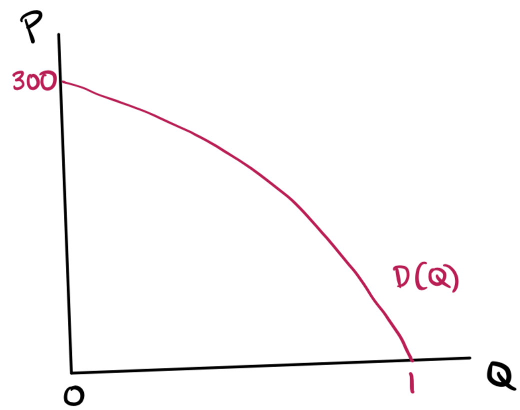
> **Solution:** Q, the proportion of people who will buy insurance, is $1 - \bar{\eta}$.
> **Grading notes:** 1 point for the right answer

iii. (2 points) Use your solutions to (i) and (ii) to solve for the consumer demand curve: In other words, find $P = D(Q)$, where Q is the fraction of individuals that will purchase at price P insurance. (Again, this will be a quadratic equation).

> **Solution:** From (ii), $\bar{\eta} = 1 - Q$. Plug this into the expression for $p(\eta)$ in (i) to solve for $p = D(Q)$:
>
> $$P = 400(1 - Q) - 100(1 - Q)^2 \rightarrow P = 100(1 - Q)(3 + Q)$$
>
> **Grading notes:** 1 point for plugging in correctly, 1 point for the right demand curve

(c) (2 points) Plot $D(Q)$ on a new graph with P and Q on the vertical and horizontal axes, respectively.

> **Solution:**



Grading notes: 2 points for correct graph.

(d) (1 point) Which way does demand $D(Q)$ slope, and why?

Solution: Demand slopes down because people with a lower probability of hospitalization have a lower willingness to pay.
Grading notes: Assuming their graph does slope down, everyone gets the point. If their graph sloped up above and they gave a reason for demand sloping up, they do not get this point.

4. Next, we derive the marginal cost of providing insurance as follows:

(a) (0 points) Remind yourself of the expected cost of providing insurance to an individual of type η.

Solution: Expected costs are 300η

(b) (2 points) Suppose a fraction Q of individuals purchase insurance at a given price. Label the marginal risk type η_Q and write an expression for η_Q in terms of Q.

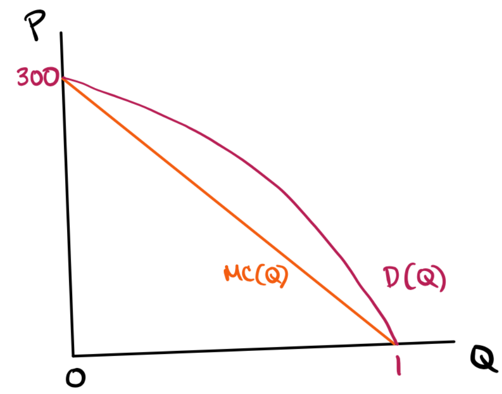
Solution: If Q people are covered, the marginal risk type is $\eta_Q = 1 - Q$
Grading notes: 2 points for correct answer

(c) (2 points) Use this to derive the marginal cost MC(Q) of insuring a fraction Q of individuals.

Solution: The marginal cost of insuring a person with risk type η_Q is $300\eta_Q$. So the marginal cost curve is $300(1 - Q)$.
Grading notes: 1 point for the marginal cost in terms of η_Q, 1 point for subbing in $1 - Q$

(d) (2 points) Plot the marginal cost curve on the same graph as you plotted the demand curve above.

Solution:



Grading notes: 2 points for correct line

(e) (1 point) What is the slope of the marginal cost curve? Why does it slope up or down?

Solution: The slope is -300. It slopes down because the marginal person has a lower risk of hospitalization than the people who are already covered. They are lower cost to insure. The people who are willing to pay the most are also the people at the highest risk of hospitalization.
Grading notes: 1 point for a reasonable explanation.

5. Finally, we derive the average cost of providing insurance as follows:

(a) (2 points) What is the average cost of insuring all individuals in the range $[\eta_Q, 1]$?

Solution: The average cost is 300 times the average value of η in $[\eta_Q, 1]$. Since η is uniformly distributed, this equals

$$\frac{300(1-(1-\eta_Q))}{2} = \frac{300(1+\eta_Q)}{2}$$

Grading notes: 2 points for the right answer, however it is derived

(b) (2 points) Use this to derive the average cost curve $AC(Q)$ of insuring a fraction Q of individuals.

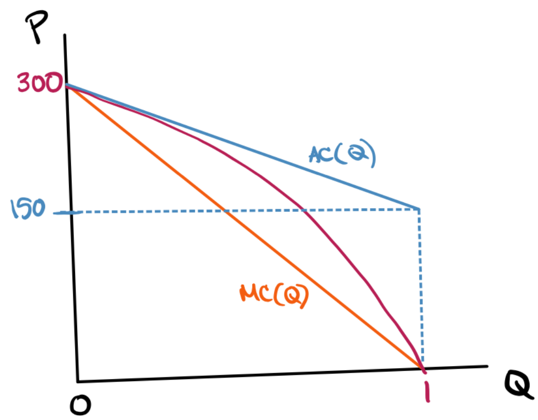
Solution: Again, sub in $Q = 1 - \eta_Q$:

$$AC = \frac{300(2-Q)}{2} = 300 - 150Q$$

Grading notes: 2 points for correct answer

(c) (1 point) Plot the average cost curve on the same graph as you plotted demand and marginal cost above.

Solution:



Grading notes: 1 point for the correct graph (-0.5 if AC curve is not everywhere above the demand curve).

(d) (2 points) What is the slope of the average cost curve? Why does it slope up or down? How does the slope of the average cost curve relate to the marginal cost curve?

Solution: The average cost curve slopes down because the marginal cost curve slopes down: if Q is small, only the highest risk individuals will be insured, and their average costs will be high. If Q is large, lower-risk people join the pool and lower the average costs. The slope of the AC curve is shallower than the MC curve because average costs won't decrease as quickly as marginal costs, since the marginal cost is averaged with a higher number to get the average cost.
Grading notes: 1 point for relating to marginal cost slope or essentially re-arguing why marginal cost slopes down. 1 point for the slope being shallower because adding lower-cost people decreases average cost but more slowly than it decreases the marginal cost.

6. Now, put those pieces together to answer the following:

(a) (4 points) Assume that there are many firms competing to offer health insurance. Therefore, the price a firm charges must equal its average cost at that price (the "zero-profit condition"). Using the demand and cost curves you derived above, compute the competitive equilibrium insurance premium $c^\star$ and quantity $q^\star$. How is this equilibrium reflected in your graph from the previous parts?

Solution: The competitive equilibrium is where $AC(Q) = P(Q)$:

$$300 - 150Q = 100(1 - Q)(3 + Q) \rightarrow 50Q(1 + 2Q) = 0 \rightarrow Q = 0, Q = -1/2 \rightarrow Q = 0$$

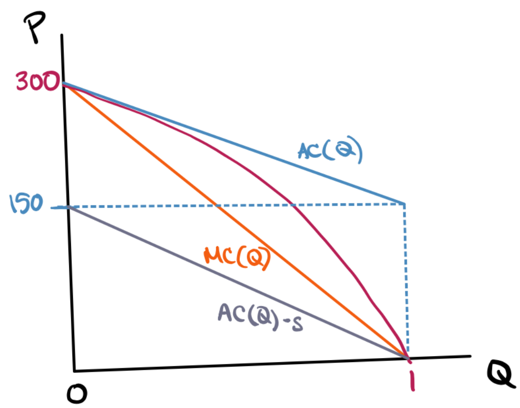
So, in the competitive equilibrium, no one is insured!
Grading notes: 2 points for setting AC=P, 1 point for correct solution, 1 point for how this is visible on graph (AC everywhere above D).

(b) (2 points) How does the competitive equilibrium differ from the socially efficient outcome? Discuss the source of the difference.

Solution: The social optimum is for everyone to receive insurance, since WTP>MC for all types. But in the competitive equilibrium, no one (except maybe the very highest type) receives insurance. This is because of adverse selection. No one is willing to pay enough to insure the average cost of everyone of their risk type and higher, which is what insurers have to charge to make zero profits when they can't identify risk types. Since insurers cannot charge different prices to different risk types (since risk types are unobservable), the market collapses.
Grading notes: 1 point for comparing the quantities, 1 point for mentioning adverse selection because insurers can't offer different contracts to different types.

(c) (4 points) Suppose the government could give a subsidy to insurance companies for each person they insure (the same subsidy for all individuals, since the government can't identify risk types either). What is the new AC curve? How big does the subsidy need to be to move the market to the socially optimal outcome? Plot the new AC curve on your supply-demand graph.

Solution: With the subsidy s, $AC(Q) = 300 - 150Q - s$. We want everyone to be insured, so we need $AC(1) = D(1)$, or $300 - 150 - s = 0 \rightarrow s = 150$



Grading notes: 1 point for writing new AC curve correctly, 1 point for realizing we need $AC(1) = D(1)$, and 1 point for the correct solution for s. 1 point for the correct graph.

(d) (1 point) Suggest one other policy that might help the market move towards the socially optimal outcome

Solution: Obvious example would be a mandate to buy coverage, or a tax on individuals who don't purchase insurance.
Grading notes: 1 point for any plausible answer

QUESTION 2: Health care potpourri (T/F/U) [20 points]

For each question, indicate whether the statement is true, false, or uncertain, and explain why, using evidence we discussed in class and in the textbook where relevant.

1. (5 points) In the past 30 years, many employers have started to offer HMO insurance plans (high deductible, low premium) rather than just PPO insurance plans (low deductible, high premium). HMOs also often contract with doctors directly, paying them a fixed salary.

Claim 1: HMOs reduce moral hazard

Claim 2: HMOs reduce the effects of adverse selection in the health insurance market

Please respond separately for Claim 1 and Claim 2.

Solution:

- Claim 1: True. When HMOs integrate insurance and medical care, they pay flat salaries to medical providers that are independent of their diagnoses and the procedures they prescribe. This reduces moral hazard effects of health insurance on provider behavior since they can't increase their incomes by increasing health care costs.
- Claim 2: Uncertain: If there was already a pooling equilibrium, having HMOs won't change the impact of adverse selection because there is already a functioning, pooling market. However, if there was not a pooling equilibrium, then introducing HMOs allows relatively healthier people take the HMO option and relatively sicker people take the PPO option, increasing the options of health insurance for low-risk people. This is a separating equilibrium, which leads patients to reveal their risk status, making the insurance market more functional and allowing both types to have actuarially-fair insurance.

Grading notes: 2.5 points each. 0.5 points for claim 1 is true, 1 point for flat salaries are independent of diagnoses and prescriptions, 1 point for this reduces moral hazard in health care costs. 0.5 points for claim 2 is uncertain, 1 point for no effect if previously pooling equilibrium, 1 point for otherwise healthier people take HMO and sicker people take PPO which leads to separating equilibrium which reveals types and sustains market.

2. (5 points) Linking health insurance to employment is inefficient.

Solution: Uncertain. There are efficiency gains from employer-provided health insurance when it reduces adverse selection in the health insurance market (by pooling risk types). There are efficiency losses from job lock (people not switching to better matched jobs because they want to keep their health insurance) and from the tax subsidy (health insurance benefits aren't taxed, unlike wages), which may raise health insurance consumption above optimal levels). Overall, the effect is uncertain.

Grading notes: 0.5 points for uncertain, 1.5 points for pooling risk types, 1.5 points for tax subsidy, 1.5 points for job lock.

3. (5 points) Optimal provision of health insurance would require that all insurance plans have first-dollar coverage, so that people are fully insured against their health costs.

Solution: False. On the margin, first-dollar coverage has low consumption-smoothing benefits, but has potentially large costs from moral hazard. The potential for moral hazard means that it is not necessarily optimal to provide full insurance because the costs of wasteful healthcare spending may outweigh these benefits. If there were no moral hazard, the small benefits of consumption smoothing would justify first-dollar coverage, but the empirical evidence from the RAND health study and other quasi-experimental work consistently finds positive elasticities of healthcare spending to insurance, making first-dollar coverage be

suboptimal for at least some people.
Grading notes: 0.5 points for 'false', 1.5 points for referencing low consumption-smoothing benefits, 1.5 points for referencing potential for moral hazard, 1.5 points for discussing the evidence on moral hazard.

4. (5 points) The individual mandate in the ACA had no effect on people who would have bought health insurance even without an individual mandate.

Solution: False. The individual mandate reduces adverse selection by encouraging healthy people with relatively low expected health costs to buy insurance. This reduces average costs to insurers. Since the ACA mandated community rating, insurers had to charge everyone the same price, and so bringing more healthy people into the market reduced the price that the insurers charged to people already in the market. Thus people who would buy without the individual mandate faced lower prices with the individual mandate, making them better off.
Grading notes: 0.5 for 'false', 1.5 points for discussing how the individual mandate reduces adverse selection, 1.5 points for discussing how this affects the market equilibrium under community rating, 1.5 points for noting that prices will fall for existing customers.

QUESTION 3: Government-provided healthcare [25 points]

There are two individuals in a society, a healthy person A and a sick person B. Healthcare is provided in this society not by insurance, but by a public healthcare system. Both people can use whatever quantity of healthcare services h_A, h_B they want free at the point of use, meaning that they don't pay any costs upfront for their healthcare. However, the healthcare services are funded by an equal tax on both individuals: $t = \frac{h_A+h_B}{2}$. It costs \$1 for the government to provide 1 unit of healthcare.

A's utility is given by

$$u_A = \ln(h_A) - t = \ln(h_A) - \frac{h_A + h_B}{2}$$

B gets more utility from healthcare, and his utility is given by

$$u_B = 3\ln(h_B) - t = 3\ln(h_B) - \frac{h_A + h_B}{2}$$

1. (3 points) How much healthcare will people choose to consume?

Solution:

$$\frac{1}{h_A} - \frac{1}{2} = 0 \iff h_A = 2$$

$$\frac{3}{h_B} - \frac{1}{2} = 0 \iff h_B = 6$$

2. (3 points) What quantities of healthcare for each person maximise a utilitarian social welfare function?

Solution:

$$SWF = \ln(h_A) + 3\ln(h_B) - h_A - h_B$$

FOCs:

$$\frac{1}{h_A} - 1 = 0 \iff h_A = 1$$
$$\frac{3}{h_B} - 1 = 0 \iff h_B = 3$$

3. (4 points) Are the socially optimal healthcare choices higher or lower than the individually chosen levels? Explain intuitively why this is the case.

Solution: Both are lower than the individually chosen levels – intuitively, people aren't paying their full marginal healthcare cost, and this fiscal causes them to over-consume healthcare relative to the social optimum.

Now suppose the government announces that people have to wait w hours per unit of healthcare they receive. Both people get disutility from waiting: the utilities of each type are now

$$u_A = \ln(h_A) - wh_A - \frac{h_A + h_B}{2}$$
$$u_B = 3\ln(h_B) - \alpha wh_B - \frac{h_A + h_B}{2}$$

Note that the two people have different disutility from waiting; if $\alpha > 1$ then B dislikes waiting more than A does, if $\alpha < 1$ then B dislikes waiting less, and if $\alpha = 1$ their disutility from waiting is equal.

4. (4 points) Find the privately optimal choices of h_A and h_B as functions of α and w.

Solution:

$$\frac{1}{h_A} - w - \frac{1}{2} = 0 \iff h_A = \frac{1}{w + \frac{1}{2}}$$
$$\frac{3}{h_B} - \alpha w - \frac{1}{2} = 0 \iff h_B = \frac{3}{\alpha w + \frac{1}{2}}$$

5. (3 points) Suppose $\alpha = 1$. Can the government can set a $w > 0$ so that the individually optimal choices of healthcare will be socially optimal? If so, what is the w that achieves this? If not, explain intuitively why not.

Solution: If $\alpha = 1$ and the government sets $w = \frac{1}{2}$, then both people have to pay a marginal cost of $w + \frac{1}{2} = 1$ for healthcare. Since this is the same as the social marginal cost of providing them healthcare, this policy achieves the social optimum.

6. (3 points) Suppose $\alpha = \frac{1}{2}$. Can the government can set a $w > 0$ so that the individually optimal choices of healthcare will be socially optimal? If so, what is the w that achieves this? If not, explain intuitively why not.

Solution: This is not possible, since the government cannot ensure that both people are paying the same marginal cost for healthcare when $\alpha \neq 1$. Choosing $w = \frac{1}{2}$ as in the previous part would cause B to consume too much healthcare, but choosing $w > \frac{1}{2}$ would cause A to consume too little.

7. (5 points) When governments are providing centralized healthcare to people in this way, they can either make people wait for access to healthcare as you analysed above, or they can charge people a fee for the healthcare they use. Without doing any more math, suggest one justification for the government making people wait for healthcare, and one justification for instead charging people a fee.

Solution: Waiting is potentially more equitable if time costs are less of a barrier for low income people than financial costs. It also might be an efficient screening mechanism if people whose healthcare we care more about (low-income people etc.) have lower waiting costs. However, waiting is wasteful, whereas a fee would generate revenue for the government which could be used to pay for healthcare and allow the government to lower taxes.