MIT 14.41 – Problem Set 6

Due December 2, 2022
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QUESTION 1: Redistribution and moral hazard [40 points]

In this problem, we will consider multiple types of redistribution in a society in which there are three types of families:

- Families of type H can earn a market wage of \$45 per hour to support three people – a working parent, a stay-at-home parent, and a child. They have a family utility function $U_H = 3ln(C) + ln(L)$.
- Families of type P can earn a market wage of \$15 per hour to support three people – a working parent, a stay-at-home parent, and a child. They have a family utility function $U_P = 4ln(C) + ln(L)$.
- Families of type S can earn a market wage of \$15 per hour to support two people – a working parent and a child, but have to pay \$10 per hour for childcare when working. They have a family utility function $U_S = 4ln(C) + ln(L)$ (but note that unlike the other families who have $C = Y$, their $C = Y - 10H$ if Y is income).

where C is all non-childcare consumption of the family which has a unit price of \$1, L is units of leisure (time spent not working) of the worker, and H is hours the worker spends working. All working parents have a maximum of 2,000 hours per year of work/leisure, and there is no saving.

1. (6 points) Calculate the choice over consumption and leisure that each family will make. What is the per-person consumption in each family?

Solution: Each family chooses L and C to maximize their utility subject to the budget constraint $C = wH = w(2000 - L)$. Specifically, families of type H solve

$$\max_L 3ln(45(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-3}{2000 - L} + \frac{1}{L} = 0 \rightarrow L = 500, H = 1500, C = 67500$$

So per-capita consumption in family H is \$22500.

Families of type P solve

$$\max_L 4ln(15(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4}{2000 - L} + \frac{1}{L} = 0 \rightarrow L = 400, H = 1600, C = 24000$$

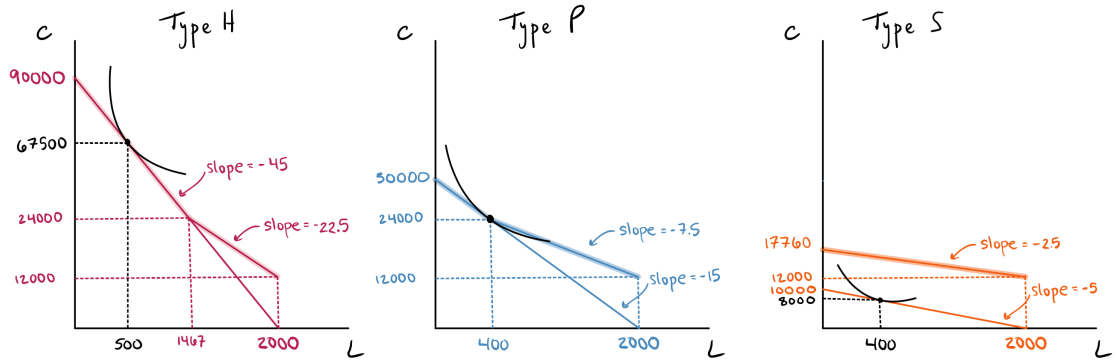
So per-capita consumption in family P is \$8000.

Families of type S solve the same problem as those of type P, but their effective wage is \$5. So they have $L = 400$, $H = 1600$, and $C = 8000$, with per-capita consumption of \$4000.

Grading notes: 2 point per family type leisure and consumption choices. Minus 1 point if do not correctly translate to per-capita consumption.

Suppose that a transfer program is introduced that provides any family a guaranteed benefit level (G) of \$12,000 with a benefit reduction rate (τ) of 0.5. Childcare spending is tax-deductible, and the benefit reduction rate is based on taxable income.

2. (6 points) Draw the budget constraints faced by each type under this new transfer program and the point at which they originally optimized their utility from the previous part. Label each point, and make sure to label any kinks in your budget constraint on both axes. (You probably want to put each type of family on a separate graph)



Solution:

Grading notes: 1 point for each original budget constraint and 1 point for each new budget constraint. Half credit if they do not label relevant points/slopes.

3. (6 points) What effect does this policy have on each family type's labor supply decision? (Hint: contrast their leisure/labor supply decisions with and without the transfer program)

Solution: Now, each family has a kinked/piecewise budget set. On the new section introduced by the transfer program, the line is $C = 12000 + \frac{w}{2}(2000 - L)$. Specifically, families of type H solve

$$\max_L 3ln(12000 + \frac{45}{2}(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-3 * 45}{24000 + 45(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 183, H = 1817, C = 40875$$

which is (1) not on this section of the budget set, and (2) which is clearly dominated by their choice they made absent the policy. This means any choice that is on the allowable portion of the budget set with the

transfer will have even lower utility. So, they keep the same L, H, and C as in part 1.

Families of type P solve

$$\max_L 4ln(12000 + \frac{15}{2}(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4 * 15}{24000 + 15(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 720, H = 1280, C = 21600$$

Comparing their utility under this choice with the utility they get if they chose absent the policy:

$$U(400, 24000 = 4ln(24000) + ln(400) = 46.3$$

$$U(720, 21600 = 4ln(21600) + ln(720) = 46.5$$

So family P chooses $L = 720$, $C = 21600$ and works less than they did before.

Families of type S solve

$$\max_L 4ln(12000 + \frac{5}{2}(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4 * 5}{24000 + 5(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 1360, H = 640, C = 13600$$

Comparing their utility under this choice with the utility they get if they chose absent the policy:

$$U(400, 8000 = 4ln(8000) + ln(400) = 41.9$$

$$U(1360, 13600 = 4ln(13600) + ln(1360) = 45.3$$

So family S chooses $L = 1360$, $C = 13600$ and works less than they did before.

Grading notes: 2 points per family type solution

4. In this part only, imagine that the benefit reduction rate was zero.

   (a) (2 points) What is the popular name for this type of redistribution policy?

   Solution: This is universal basic income.

   Grading notes: 2 points for UBI

   (b) (6 points) Draw another graph with each type's original budget constraint and their new budget constraint under this policy. Relative to their original labor supply decisions, what effect does this policy have on each type's labor supply decision? Add the families' new leisure-consumption choices to your graphs. (Again, you should put each family type on a different graph).

   Solution: Now, each family maximizes their utility subject to the constraint that $C = 12000 + w(2000 -$

L) for all L. Specifically, for families of type H,

$$\max_L 3ln(12000 + 45(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-3 * 45}{12000 + 45(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 567, H = 1433, C = 76500$$

For families of type P,

$$\max_L 4ln(12000 + 15(2000 - L)) + ln(L)$$

Taking FOCs:

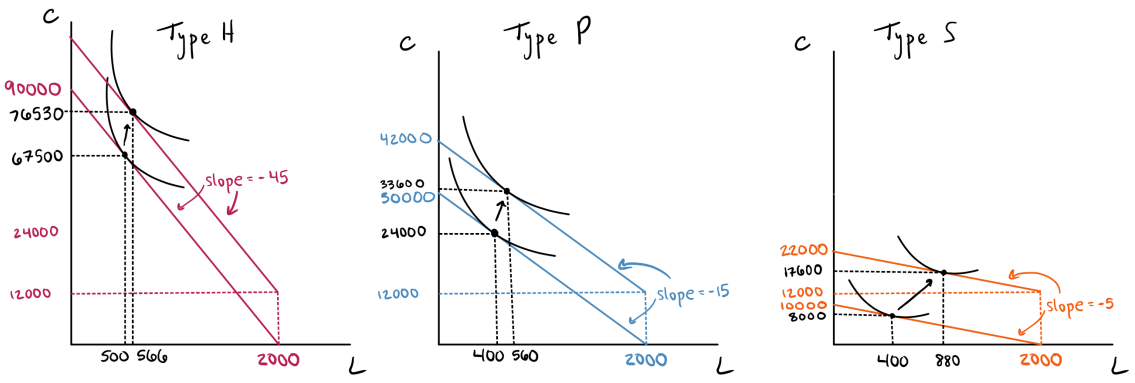
$$\frac{-4 * 15}{12000 + 15(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 560, H = 1440, C = 33600$$

And for families of type S,

$$\max_L 4ln(12000 + 5(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4 * 5}{12000 + 5(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 880, H = 1120, C = 17600$$



Grading notes: 1 point for each family type's new choice of consumption and leisure, 3 points for the correct graphs.

(c) (5 points) How does the effect of the policy on labor supply change when $\tau = 0$ compared to when $\tau > 0$, and why?

Solution: For families of type P and S, When $\tau = 0$, there are only income effects (people are richer, so they work less, since leisure is a normal good). When $\tau > 0$, there is an income effects plus a substitution effects: leisure is relatively less expensive, and people work even less than when $\tau = 0$. For families of type H, there is no income or substitution effect when $\tau > 0$ because they are unaffected by the policy, but an income effect when $\tau = 0$, so they work less when $\tau = 0$, the opposite of the other families.

Grading notes: 3 points for correct discussion of income and substitution effects. 2 points for P and S work less when $\tau > 0$ than when $\tau = 0$, 1 point for H works more when $\tau > 0$ than $\tau = 0$.

(d) (2 points) Why might the government prefer to have $\tau > 0$? Why might they prefer $\tau = 0$?

Solution: When $\tau = 0$, everyone gets a transfer. Especially if the government cares about getting families above some absolute level of poverty, they do not want to give a transfer to the H type families. However, $\tau > 0$ distorts the labor supply decision, especially for families who earn low wages. While $\tau = 0$ does change the labor supply decision of families relative to the world with no policy, it does not implicitly tax wages, meaning that workers are still efficiently trading off between leisure and consumption (it's just that at a higher basic income level, they get more value from a bit more leisure).

Grading notes: 1 point for $\tau = 0$ gives money to high-income families, 1 point for $\tau > 0$ distorts the labor supply decision.

5. In this part only, suppose that the government instead provides a child tax credit of \$15 per hour worked that is restricted to single-parent families.

(a) (3 points) Relative to their original labor supply decisions, what effect does this policy have on each family type's labor supply decision and consumption?

Solution: Families of type H and P are unaffected, because they are not single-parent families. Families of type S face the same problem as in part 1, except their effective wage is now \$20 (\$15-\$10+\$15). So, they choose L = 400/H=1600 as above and have consumption of \$32,000.

Grading notes: 1 point for families of type H and P being unaffected, 1 point for single parents now having an effective wage of \$20, 1 point for the correct labor supply and consumption.

(b) (4 points) Besides the effects on labor supply you found above, what are two other economic reasons the government might prefer this policy to their original transfer program?

Solution: On reason could be that they care particularly about helping single-parent families (i.e. if the social returns to increased consumption are larger for single-parent families, or if the social welfare function has a larger weight on these families, or if the society cares about everyone having equal opportunities to work regardless of child-having and marital status). Another reason is that they may think that being in high-quality formal child care/preschool may be beneficial for these low-income children's long run outcomes (consistent with evidence discussed in the textbook).

Grading notes: 2 point per reason, full credit for any plausible economic (not political) reason.

QUESTION 2: Place-based redistribution policy [30 points]

The newly-elected governor of Massachusetts wants to reduce inequality in the state. To simplify the problem, assume that there are two cities in the state: City W is relatively wealthy, and City P is relatively poor. City W has residents who earn a higher average income, its workers are more productive due to substantial public and private investment, its schools are better-funded through local taxes, and its crime rates are lower.

1. (7 points) At first, Gov. Healey is considering two proposals to achieve her goal. One proposal involves making the tax rate more progressive: lowering the marginal tax rate for the first dollars of income, and raising the

marginal tax rate for those at the top of the income distribution. The second proposal is place-based: it involves collecting a lump-sum tax from people living in City W and giving a cash grant to each person living in City P.

In terms of efficiency and redistribution, discuss the pros and cons for each policy, making sure to describe what must be true for the income tax change to be a better policy choice than the place-based tax-and-grant system and vice versa.

Solution: Note: throughout this problem, be generous with partial credit for thoughtful and correct answers that do not exactly follow the line of reasoning in the solutions.

Since most residents in city P are poorer than those in city W, location acts as a tag that aids redistribution. Location is easily observable, and if moving costs (pecuniary or non-pecuniary) are high, people may not move in response to the policy, reducing moral hazard costs and causing only small efficiency consequences relative to changes in the income tax – modifying the place-blind income tax schedule may have negative labor supply consequences among the better off in either city.

Alternatively, if moving costs are low, the grant may induce some residents of city W to move to city P, which could have substantial negative efficiency consequences if the workers in these households are less productive in city P or if their children receive worse educations in the under-funded city P school system. Moreover, moving erodes the tax base in city W, so it may necessitate a smaller cash grant and therefore less redistribution in the end.

Grading notes: 1 point for location tagging for redistribution, 3 points for the case where moving costs are high (1.5 points for no moving = no moral hazard, 1.5 points for income tax change may have labor supply inefficiencies), 3 points for the case where moving costs are low (1.5 points for negative efficiency effects if people move to city P, 1.5 points for this might also reduce redistribution).

2. (5 points) A group is lobbying against the cash grant policy, arguing that it is unfair. Give one argument that they could make, relating the properties of the policy to the equity concepts discussed in class.

Solution: The grant may violate horizontal equity: a poor household that happens to live in city W will not get the grant, while a similarly poor household in city P will. The poor household in city W may be better off because of non-pecuniary benefits (better schools, less crime, etc.) of living in city W, but even after adjusting for this, but if we assume that there is substantial luck involved in living in city P or city W, we still think this is violating horizontal equity. (If there are no moving costs or credit constraints, where you live is a choice and we don't think there is any horizontal inequity). It will also violate vertical equity, if both rich and poor people in city W pay the tax and both rich and poor people in city P get the lump sum.

Grading notes: 1.5 points for mentioning horizontal equity, 2 points for explanation of horizontal equity, 0.5 points for why we are still worried even if living in city W delivers non-pecuniary benefits. 1 point for vertical equity.

3. (7 points) Another group is lobbying for a different policy: Instead of an unconditional cash grant, residents of city P would receive a moving subsidy if they decide to move to city W, again to be funded by a tax on existing

residents of city *W*. Give one argument that they could make, explaining conditions under which the subsidy will have a positive effect on efficiency. What does the evidence discussed in class suggest about what the effects of this policy might be?

Solution: A one-time moving subsidy could have long-term benefits if the workers who move become much more productive in city *W*, or if their children receive better educations in the city *W* school system. The return on the government's investment here may be high, so that the policy is not only redistributive but also efficiency-enhancing. Note that this requires that residents in city *P* face both moving costs and credit constraints that prevent them from borrowing against higher future earnings to make the move – without both of these, the subsidy will not be efficiency enhancing and will only introduce a new distortion (DWL) into the economy.

The evidence we've seen on this type of subsidy comes from the Moving to Opportunity (MTO) experiment. That evidence suggests that the return on investment may be very high, but takes a generation to appear: the children who move to City *W* when they are young will earn more when they grow up, but parents who move realize minimal immediate benefits (some positive health effects).

Grading notes: 2 points for the argument that workers who move may be more productive or their children may get better educations, 2 points for this can only be efficiency-enhancing if there are moving costs and credit constraints (1 point each), 1 point for mentioning MTO, and 2 points for discussing the relevant results from MTO.

4. (5 points) Another group is lobbying against the moving subsidy on redistributive grounds, arguing that many people can be made worse-off by the policy. Give one argument that they could make, relating the properties of the policy to the equity concepts discussed in class, and discuss how this may go against Gov. Healey's original goal.

Solution: If the subsidy is not sufficiently generous, the poorest households in city *P* may be left behind. This is vertically inequitable – the moving subsidy does not fall with income. With fewer local tax revenues to fund schools, and likely smaller investment by the local government and private firms, the remaining residents of city *P* may be left with lower wages and a worse school system. This may persistently lower the well-being of these residents, exacerbating inequality in Massachusetts.

Grading notes: 2.5 points for discussing vertical equity and correctly applying it, 2.5 points for discussing residents left behind/negative effects of people moving out on city *P*.

5. (6 points) A final group is arguing that the government should instead use the tax revenue from City *W* to provide a free job-training program for the residents of City *P* and to provide grants to the schools in City *P*, so that City *P* becomes more like City *W* in terms of investment and infrastructure. On theoretical grounds, why might this be less preferred than the policies discussed so far? What must be true in order for this policy to make those in City *P* better off? Empirically, what does the evidence suggest about the effects of job training and school spending?

Solution: By providing in-kind rather than cash transfers, the government is imposing its preferences on citizens in city P, who may have other needs that they would meet if they received a cash transfer. In particular, they may have short-term needs (food, housing, etc.) that are not addressed by a program that relies on long-term improvements in human capital to increase incomes. In other words, this policy imposes the government's preferences on the residents of City P, who may not have chosen to spend an income increase on this type of investment. In addition, in order for those policies to make those in City P better off, job training and school spending have to actually improve economic outcomes. Empirically, the evidence suggests that job training has positive, but small, effects on earnings and that increased school spending does improve test scores and graduation rates.

Grading notes: 1 point for mentioning in-kind versus cash transfers, 1 point for how this is paternalistic/imposes government preferences/may not meet immediate needs. 2 points for in order to work, job training and school spending have to improve outcomes. 2 points for correct discussion of empirical evidence.

QUESTION 3: Incidence of a Pigouvian tax [30 points]

Suppose that the total private marginal benefit to consumers from using Q gallons of gasoline is

$$PMB(Q) = Q^{-\frac{1}{\alpha}}$$

where $\alpha > 0$ is a constant. The marginal cost of supplying Q gallons of gas is

$$PMC(Q) = Q^{\frac{1}{\beta}}$$

where $\beta > 0$ is a constant. We assume that the market is competitive. This means that producers supply gas so that the price equals the private marginal cost, and consumers demand gas so that the price equals the private marginal benefit:

$$D(P) = P^{-\alpha}, S(P) = P^{\beta}$$

In equilibrium, $D(P) = S(P) = Q$.

However, consuming Q gallons of gas produces carbon emissions of E kg, which has negative externalities that producers and consumers of gas do not take into account. The EPA estimates that each gallon of gas consumed produces 8.887 kilograms of carbon emissions, $E = 8.887Q$, and that the social cost of these emissions is constant at 5.1 cents per kilogram.

1. (3 points) Calculate the optimal per-unit (per-gallon) Pigouvian tax on gasoline.

Solution: Since the marginal externality is constant, the optimal Pigouvian tax is just equal to this marginal externality: $8.887 \times 5.1 \times 10^{-2} = \$0.453$

Grading notes: 1 point for identifying that optimal tax = marginal externality, 1 point for calculating marginal externality by multiplying carbon kg per gallon by social cost per kg, 1 point for correct numbers.

2. (4 points) Calculate the demand elasticity η_D and the supply elasticity η_S of gasoline.

Solution: $\eta_D = \frac{dD}{dP}\frac{P}{D} = -\alpha P^{-\alpha-1}\frac{P}{P^{-\alpha}} = -\alpha$; similarly $\eta_S = \frac{dS}{dP}\frac{P}{S} = \beta P^{\beta-1}\frac{P}{P^{\beta}} = \beta$.

Grading notes: For each elasticity, one point for getting correct formula and one point for correctly simplifying to the constant elasticity form.

3. (5 points) Suppose that consumers pay a tax of τ per unit of gas they buy, so producers receive P per gallon while consumers pay $P + \tau$. Calculate:

   (a) The change in the pre-tax price

   (b) The change in the price consumers have to pay for gas, including the tax

Solution: Standard incidence formula:

$$\Delta P = \frac{\eta_D}{\eta_S - \eta_D}\tau = \frac{-\alpha}{\alpha + \beta}\tau$$

is the change in the pre-tax price, and the change in the post-tax price is

$$\tau + \Delta P = \tau - \frac{\alpha}{\alpha + \beta}\tau = \frac{\beta}{\alpha + \beta}\tau$$

Grading notes: 1 point for writing down correct formula (in elasticity terms) for pre-tax price change, 1 point for post-tax price change formula, 2 points for substituting in elasticities correctly to formula, 1 point for both expressions correct.

4. (3 points) Suppose that $\alpha = 0.1$, $\beta = 1$. What percentage of the burden of the tax is borne by consumers, and what percentage by producers?

Solution: Plugging in numbers, we get

$$\Delta P = -\frac{0.1}{1.1} \approx 0.091, \tau + \Delta P = \frac{1.0}{1.1} \approx 0.909$$

so that consumers bear 90.9% of the burden of a \$1 tax increase and producers bear 9.1%.

Grading notes: 1 point for substituting in appropriately into each expression (2 total), 1 point for correct numerical values.

5. (2 points) Suppose the equilibrium price of gas without a tax is \$3.50 per gallon. If the tax is set at the optimal level you calculated in part 1, and $\alpha = 0.1, \beta = 1$, calculate the new pre-tax and after-tax prices of gas (rounded to the nearest cent).

Solution: Plugging in numbers, we get that the pre-tax price will fall by \$0.04 to \$3.46, and the post-tax price will rise by \$0.41 to \$3.91.

Grading notes: 1 point for correct pre-tax price, 1 point for correct post-tax price.

6. (4 points) A politician complains that consumers are bearing too much of the burden of gasoline taxes, and argues that instead producers should be made to pay a tax τ for each unit of gas that they sell, so that producers receive $P - \tau$ for every gallon of gas while consumers pay P. Show that this would have no impact on who bears the burden of the tax.

Solution: With a producer tax, we instead get

$$\Delta P = \frac{\eta_S}{\eta_S - \eta_D}\tau = \frac{\beta}{\alpha + \beta}\tau$$

so the consumer price again changes by

$$\frac{\beta}{\alpha + \beta}\tau$$

while the producer price is again

$$\Delta P - \tau = \frac{\beta}{\alpha + \beta}\tau - \tau = -\frac{\alpha}{\alpha + \beta}$$

Since neither changes, burden of the tax is identical.

Grading notes: 2 points for correctly showing that consumer price is unchanged, 2 points for correctly showing that producer price is unchanged. 1 point for either if argument is incomplete. 2 points at most if they just appeal to the result from lectures that tax burden does not depend on the side of the market without directly showing that prices are unchanged.

7. (4 points) Another politician, who has a hazy memory of taking a class in public finance when he was in college many years ago, argues that the tax should be abolished completely, because it is distorting the market for gasoline and causing a deadweight loss. Comment on this argument.

Solution: Two possible comments: first, it might still make sense to raise a tax that has a deadweight loss if there are no non-distortionary ways to raise revenue; second, in this case the tax is good for social welfare, because it is correcting an externality, and so reduces rather than increasing deadweight loss.

Grading notes: Give full credit for *either* of these comments if well-argued, or an alternative valid comment. Partial credit if the argument is incomplete.

8. (5 points) The analysis above is in partial equilibrium, and considers the market for gasoline in isolation. If we did a complete general equilibrium analysis, suggest and explain one possible reason why part of the incidence of the tax might be borne by people who do not own a gas-powered vehicle and never buy gasoline (i.e. people who do not participate in the market we analysed above).

Solution: Any answer that explains a spillover onto a different market where non-gas car users participate is valid. Some examples are:

- A gas tax might raise demand for electricity if people switch to electric cars, raising electricity prices for everyone

- A gas tax might raise demand for public transit and increase the cost of public transit (if gas is required for e.g. buses), thus raising prices for everyone

Grading notes: 2 points for identifying an alternative market that could be affected, 1 point for identifying a spillover that *raises* prices in that other market, and up to 2 points for explaining how the spillover would work.

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