Active learning Session 2

We saw in class that the torus $S^1 \times S^1$ is homeomorphic to the quotient of the square $[0,1] \times [0,1]$ illustrated in Figure 1. How about other surfaces ?

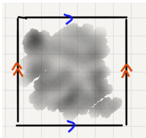

FIGURE 1 – The torus as a quotient of $[0,1] \times [0,1]$.

Problem 1

Describe the 2-sphere and the Möbius band as quotients of $[0,1] \times [0,1]$. Don't write formulas : just draw the identifications as we did for the torus.

Problem 2

Try to understand the quotient in the left hand side of Figure 2 and why it is the Klein bottle.

Problem 3

Try to understand the quotient on the right hand side of Figure 2 and why it is the "compact orientable surface of genus 2" (aka the "hollow doughnut with two holes Σ_2"). Generalise this to g holes.

Try to come up with the definition of the operation # known as *connected sum* such that $\Sigma_{g+h} \cong \Sigma_g \# \Sigma_h$.

Problem 4

Try to understand the quotient illustrated in the central picture of Figure 2 and why it is the real projective plane Compare this with the description mentioned on the third problem set.

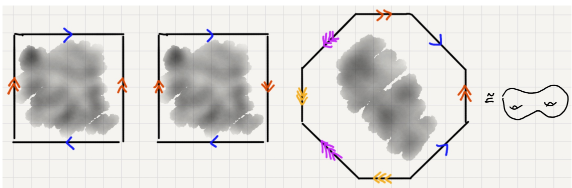

FIGURE 2 – Some familiar quotient spaces.