Active learning Session 4

We learn some more about path-connectedness and its relation to connectedness. Along the way, we obtain further examples of connected spaces.

Problem 1

A subset of $X \subset \mathbb{R}^n$ is called *convex* if for every $x, y \in X$ and every $t \in [0, 1]$, we have $tx + (1 - t)y \in X$. Illustrate this concept with a sketch and show that convex sets are path-connected.

Problem 2

For $n > 1$, show that $\mathbb{R}^n \setminus \{0\}$ is path-connected.

Problem 3

Write S^k for the unit sphere in $\mathbb{R}^{k+1}$:

$$S^k = \{(x_1, \ldots, x_{k+1}) \in \mathbb{R}^{k+1} \mid x_1^2 + \ldots + x_{k+1}^2 = 1\}.$$

Prove that $\mathbb{R}^n \setminus \{0\}$ is homeomorphic to $S^{n-1} \times \mathbb{R}_{>0}$ (you should write a formula for a homeomorphism, but you need not verify continuity in full detail). Deduce that the sphere S^n is path-connected for $n > 0$.

Problem 4

For $n > 1$, prove that $\mathbb{R}$ and $\mathbb{R}^n$ are not homeomorphic.

Mini-Lecture

We saw in class that if a topological space is path-connected, then it is connected. Time permiting, we discuss a partial converse.