Active learning Session 5

Problem 1

Describe sequential compactness in spaces endowed with the trivial and discrete topology.

Problem 2

Prove that if X is a compact metric space, then it is sequentially compact.

Hint : First, use compactness and reason by contradiction to show that every sequence (x_n) in X admits a *limit point* i.e. there exists $y \in X$ such that for every open set $U \subset X$ containing y, infinitely many of the x_n belong to U. Then use the neighborhood basis $\{B(y, \frac{1}{k})\}_{k \geq 1}$ of y to find a subsequence of (x_n) that converges to y.

Mini-Lecture

We state a result generalising problem 2 and discuss the converse.