Problem Set 0

This is not really a problem set : there is nothing to turn in! The goal is simply to highlight some concepts that will be used early on in the class.

Problem 1

Make sure you are comfortable with the following notions from set theory : unions, intersections, subsets, power sets, complements, [1] images, inverse images, injective maps, surjective maps and bijective maps. Inverse images will be particularly important to the class.

If you are uneasy with any of these notions, do not hesitate to ask about them during an office hour.

Problem 2

Read the following definition :

Definition 1 A *group* $(G, \cdot)$ is the data of a set G together with a *group law* $\cdot : G \times G \to G$ that satisfies the following three axioms :

1. the group law is associative : $g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$ for every $g_1, g_2, g_2 \in G$;
2. the group law admits an element e_G, known as the *identity element* such that $g \cdot e_G = g$ and $e_G \cdot g = g$ for every $g \in G$; [2]
3. for every $g \in G$, there is an element $h \in G$, known as *the inverse of* g, such that $g \cdot h = e_G$ and $h \cdot g = e_G$. [3]

In practice, one often writes e instead of e_G as well as xy instead of $x \cdot y$. Additionally, the inverse of g is denoted g^{-1}. Also, because of the associativity axiom, one writes xyz without any parentheses.

In the second part of the class, we will need some notions from group theory (we will after all be studying the so-called fundamental *group*!). As group theory is not a prerequisite to this course, the last exercise of each problem set will introduce a concept from group theory.

1. For instance, make sure you are familiar with *de Morgan's law :* If A, B are subsets of a set X, then $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$ and $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.
2. The identity element is unique : if e_G, e'_G are identity elements, then $e_G = e_G \cdot e'_G = e'_G$.
3. The inverse of g is also unique : if h_1, h_2 are inverses of g, then

$$h_1 = h_1 \cdot e_G = h_1 \cdot (g \cdot h_2) = (h_1 \cdot g) \cdot h_2 = e_G \cdot h_2 = h_2.$$