Problem Set 11:

Covering spaces

Problem 1

Give two examples of finite connected covers of $S^1 \vee S^1$ (these examples must be non-trivial and different from the one given in class) and one example of an infinite connected cover of $S^1 \vee S^1$.

You don't have to write detailed proofs, just draw a sketch of the covers and explain what the covering map does. Describe the generators of the fundamental groups of your covers using the fact that "if X is a connected graph with maximal tree T, then $\pi_1(X)$ is free on the number of edges of $X \setminus T$."

Problem 2

Give an example of a connected covering space of Σ_2 with an action of

1. $\mathbb{Z}_3$ by homeomorphisms ;
2. $\mathbb{Z}$ by homeomorphisms ;
3. $\mathbb{Z}^2$ by homeomorphisms.

You don't have to write detailed proofs, just draw a sketch of the covers and explain what the covering map and the action do.

Problem 3

Recall that the *abelianisation* of a group G is the group $G^{\text{ab}} = G/[G, G]$, where $[G, G] = \langle [g, h] \mid g, h \rangle$ is normal and is called the *commutator subgroup* ; recall also the notation $[g, h] := ghg^{-1}h^{-1}$.

1. Prove that if $\langle X \mid R \rangle$ is a presentation of a group G, then its abelianisation is presented by $\langle X \mid R \cup \{[x, y] \mid x, y \in X\} \rangle$.
2. Deduce that Σ_g and Σ_h are not homotopy equivalent for $g \neq h$.